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Entrepreneurship and Income Distribution Dynamics: Why Is the Income Share of Top Income Earners Acyclical over the Business Cycle?*

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Abstract

The U.S. economy shows a contrasting difference in cyclical properties between low- and high-income groups. Income shares of the bottom three income quintiles are procyclical, while those of the next 35 percent are countercyclical. However, the income share of the very top five percent income group is acyclical over the business cycle. This study attempts to explain the cyclical behavior of the income distribution over the business cycle, particularly focusing on the top five percent income earners' share, using a heterogeneous agent model featuring a choice to become an entrepreneur. The model economy successfully reproduces the acyclical behavior of the income share of the top five percent income earners as well as the overall cyclicality of income shares of the income groups. In an economic expansion, relatively more people in the low income groups newly begin employment as workers, which raises the income shares of the bottom three quintiles and lowers the income shares of the top two quintiles, while at the top, relatively more people become entrepreneurs, which offsets the decline in the income share of the high-income earners from the workers' side.

JEL classification: E32; C68; D31

Keywords: Income distribution dynamics; Entrepreneurship; Occupational Choice

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1 Introduction

The "Occupy Wall Street" movement drew much attention to the inequality of distribution, in particular, to the concentration of wealth and income in the top one percent. It may be surprising that Piketty (2014)'s 700-page book, *Capital in the Twenty-First Century*, was listed as a best-seller on Amazon, but it reflects a lot of interest in the inequality of income and wealth. The long-run trend that income or wealth distribution becomes more unequal motivated many researchers to identify the causes and effects of the concentration of wealth and income. In fact, existing papers in the literature have devoted a great deal of effort to studying the causes of the highly concentrated distributions,[1] but most of them have mainly focused on reproducing the steady state distribution of income or wealth. In addition to the level and long-run trend of inequality, the issue of income distribution dynamics over the business cycle is also central to the discussion of economic policies, such as redistributive as well as stabilization policies. In spite of its importance, replicating the cyclical behavior of income distribution is in its infancy in that not only there are only a few studies on this issue, but also the conventional models trying to explain the business cycle dynamics of income distribution have had limited success. This study is another attempt to explain the business cycle behaviors of income distribution with an emphasis on the top income earners.

In the U.S. economy, there is a significant difference in the cyclical properties of the low and high-income groups. As found in Castañeda, Díaz-Giménez, and Ríos-Rull (1998), the income shares of the first three lowest income quintiles are strongly procyclical over the business cycle, strongly countercyclical for the fourth quintile and the 80-95 percentile, and, interestingly, acyclical for the top five percent.[2] That is, during expansions, the income shares of the individuals in the low-income quintiles tend to increase, as new employment occurs mostly among the low income, while those of the individuals in the high-income groups tend to decrease. However, the income share of the top five percent group does not show a cyclical behavior, basically unaffected by the business cycle. These empirical findings are quite surprising and interesting in that individuals in the top income groups not only earn a high portion of the total income, but their income share is also steady over the business cycle. This is a very desirable feature of low risks and high returns, contrasting with the conventional wisdom of high income coming along with high risks.[3] We propose a hypothesis

[1] For example, Cagetti and De Nardi (2006) and Quadrini (2000) successfully replicate the unequal wealth distribution by incorporating entrepreneurship into heterogeneous agent models, and Heathcote, Perri, and Violante (2010) investigate an empirical analysis of various dimensions of inequality in the U.S.

[2] We use the Current Population Survey (CPS) data for the period of 1968-2016, while Castañeda, Díaz-Giménez, and Ríos-Rull (1998) use the CPS samples from 1948-86.

[3] We do not talk about micro-level risks but aggregate macro-level risks of the income groups over the business cycle.

that entrepreneurial activities play an important role in the cyclical behavior of the income shares for the top income earners. To support this conjecture, we first empirically document the large income share of entrepreneurs in the top five percent, compared to that of the fourth quintile or the 80th to 95th percentile group, with the large population share and the high average income of entrepreneurs relative to workers. We also find empirical evidence that transition between workers and entrepreneurs is very active over the business cycle, in particular, from workers to entrepreneurs in booms in the top income group. More households work as workers or entrepreneurs in booms, but a relatively large portion of workers switch to entrepreneurs in the top income group, which offsets a decline in the income share of the top income earners from the workers' side since entrepreneurs earn higher income than workers on average. Hence, this transition activity between entrepreneurs and workers may produce the acyclical behavior of the income share of the top income earners.

The purpose of this study is to replicate the cyclical behavior of the income distribution over the business cycle, focusing on the behavior of the income share of the top five percent income earners. To this end, we incorporate an entrepreneurial choice in a heterogeneous agent model. Specifically, we build a simple dynamic stochastic general equilibrium (DSGE) model featuring a large population of households who face idiosyncratic risks to labor efficiency and entrepreneurial productivity. As in Huggett (1993) and Aiyagari (1994), the capital markets are incomplete as households cannot fully insure against the two idiosyncratic shocks. The market incompleteness, together with borrowing constraints, helps produce substantial heterogeneity across individual households according to their income and wealth. One of the main features in our model is an occupational decision for households to become entrepreneurs, as in Quadrini (2000), Cagetti and De Nardi (2006), and Terajima (2006). In the model economy, households make an occupational decision among three occupational choices: employment, entrepreneurship, and no work. A household may decide to provide labor supply to the market as a worker, to run its own business by investing its labor and capital as an entrepreneur, or not to work due to unfavorable individual labor productivity shocks and/or low entrepreneurial productivity shock.[4] Relative to the previous studies on entrepreneurship in the literature, another important feature is that our model economy embeds aggregate productivity shocks in addition to the two idiosyncratic shocks in an occupational choice model. In this sense, our model is an extended version of the traditional heterogeneous agent models of Krusell and Smith (1998) and Chang and Kim (2007), who introduce aggregate uncertainty in an incomplete market model. Incorporating aggregate productivity shocks allows for fluctuations in the income distribution over the business cycles so that we can investigate the cyclical behav-

[4]The previous studies consider either an occupational decision between the two choices of workers and entrepreneurs (e.g. Quadrini (2000) and Cagetti and De Nardi (2006)) or a decision between the two choices of workers and non-employed workers (e.g. Chang and Kim (2007)).

ior of the income distribution. To the best of our knowledge, this study is the first quantitative work studying a heterogeneous agent model with occupational choices in the presence of aggregate shocks.

The main findings of this study are summarized as follows. The transition between working households and non-employed workers has been found to be important in reproducing the overall cyclicality of income shares over income groups (Castañeda, Díaz-Giménez, and Ríos-Rull, 1998). Importantly, we find that introducing a decision to become entrepreneurs plays a crucial role in reproducing the acyclical behavior of the income share of the top income earners. During expansions, there are more opportunities for entrepreneurship, especially for the top income earners, which induces more individuals to become entrepreneurs and earn high income. This offsets a decline in the workers' income share in the top income group, caused by an increase in the employment of workers in the low-income quintiles.

Related Literature This paper is related to two strands of the heterogeneous agent DSGE literature: the cyclical behavior of income distribution and entrepreneurial activities. The work of Castañeda, Díaz-Giménez, and Ríos-Rull (1998) is the first comprehensive study attempting to account for the income distribution dynamics over the business cycle in a dynamic general equilibrium model with infinitely-lived agents and unemployment risk. They document the income distribution dynamics and introduce unemployment spells and a cyclically moving labor share to account for the cyclical behavior of the income distribution. They find that unemployment spells may play a significant role in the income distribution dynamics, which is supportive of the indivisibility of labor, but they do not find the role of a cyclically moving labor share in explaining the cyclical properties of such dynamics. Even though they are successful in generating the right signs of the contemporaneous correlations between the income shares and output for most of the income groups, the correlations are very high in absolute terms, and the income share earned by the top five percent income group shows a strong negative correlation with output, though acyclical in the data. Heer (2013) also tries to explain the cyclical behavior of the income distribution with a model of overlapping generations and the savings motive for retirement, but he finds that the model economy with wage rigidity does not improve the cyclical behavior of the income distribution over Castañeda, Díaz-Giménez, and Ríos-Rull (1998), in particular, the acyclical property of the income shares of the top five percent. The contribution of this paper to the literature is that the model with a choice to become an entrepreneur successfully reproduces the salient features of the acyclical behavior of the income share of the top five percent income earners as well as the overall cyclicality of income shares of the income groups.

The second strand of the previous quantitative studies closely related to our work is the entrepreneurial choice models by Quadrini (2000), Cagetti and De Nardi (2006), and Terajima (2006). Quadrini (2000) constructs a dynamic general equilibrium model that incorporates an entrepreneurial choice and financial frictions under incomplete markets.[5] He shows that the model economy can generate a high inequality in wealth distribution and replicate the empirical patterns of wealth mobility. Cagetti and De Nardi (2006) also study a model economy with an occupational choice allowing for entrepreneurial activities, similar to Quadrini (2000), to analyze the effects of borrowing constraints as a source of wealth concentration. In contrast to Quadrini (2000), who introduces exogenous financial frictions, Cagetti and De Nardi (2006) construct endogenously determined borrowing constraints in equilibrium and find that the model economy sufficiently reproduces the observed wealth distribution for entrepreneurs and workers, and more restrictive borrowing constraints generate less wealth concentration. Terajima (2006) also studies a model considering relations between occupational and educational choices to analyze the interaction between the changes in earnings and wealth inequality over time and finds that the model can explain about 33 percent of the change in the relative average wealth between different education-occupation groups. This study contributes to the literature by introducing aggregate productivity shocks to an occupational choice model to examine the importance of entrepreneurs' behavior in business cycle properties of income shares for the top income earners.

This paper is organized as follows. Section 2 summarizes some empirical facts on the importance of entrepreneurship over the income distribution and the cyclical behavior of the income distribution. Section 3 introduces a heterogeneous agent model economy with three occupational choices. Section 4 examines the simulation results, with regard to steady state distributions and cyclical properties, focusing on the cyclical behavior of income distribution. In Section 5, we characterize how a change in the aggregate productivity affects the selection into employment and entrepreneurship based on a simple static model and discuss the role of the collateral constraints in the income distribution dynamics. Section 6 summarizes the findings and concludes.

2 Empirical Facts

In this section, we summarize some empirical evidence on the income distribution dynamics, particularly focusing on the behavior of entrepreneurs over the business cycle. We first document the distribution of entrepreneurs across income groups and then investigate the relationship between

[5] Yurdagul (2017) also constructs a model economy which features a flexible hours motive for entrepreneurship and finds that the model can generate the income difference between entrepreneurs and comparable workers.

entrepreneurship and the cyclical behavior of the income shares earned by the top income earners. Our analysis is based on the two data sets: the Current Population Survey (CPS) and the Panel Study of Income Dynamics (PSID).

2.1 Data

We use the Annual Social and Economic (ASEC) supplement of the CPS, which contains detailed questions covering economic characteristics surveyed every March. The basic unit of observations for the CPS is a household, and the sample size is around 65,000. The CPS data samples used in this study span from 1968 to 2016 (49 years), which are downloaded from the Integrated Public Use Microdata Series (IPUMS). We define a household income as the sum of labor income, self-employment income, and net asset income, but we exclude private transfers, public transfers, and taxes to be consistent with the definition of income in the model economy.[6]

Since this paper focuses on the behavior of the top income households, top-coding is an important issue. In the CPS, the thresholds of top codes vary across income sources and across time. For the years prior to 1996, income in the CPS was top-coded at the values between 50,000 and 99,999. For these survey years, we multiply the top-coded income values by an adjustment factor of 1.5 as in Katz and Murphy (1992), Autor, Katz, and Kearney (2008), and Autor and Dorn (2013). Starting in 1996, the top-coded individuals were divided into twelve groups depending on characteristics such as gender, race, and full time status, and income values of top-coded individuals were replaced with the mean income within each group. In 2011, the Census Bureau shifted from the average replacement value system to a rank proximity swapping procedure. We use the Census Bureau data unadjusted for the years starting from 1996 since we are interested in the behavior of the top five percent as a group rather than as an individual, and our work is not subject to the treatment of the Census Bureau regarding top-coding issues.

We classify a household by occupational status into three categories: entrepreneurs, workers, and non-employed workers. Following Quadrini (2000), we define a household as an entrepreneur when at least one person in a household earns self-employment income. This implies that the entrepreneurship of anyone in a household is enough to include the whole household in the business group.[7] In the CPS, there is a question indicating whether the respondent worked in the preceding week. With this question we define workers and non-employed workers: a worker is a household,

[6] See Appendix for more details on the sample selection and the definitions of each income category.

[7] As a robustness check, we define entrepreneurs as households with self-employment income above the threshold income between the first and second income quintiles to ensure that entrepreneurs are not mostly unemployed that become self-employed, and we find no meaningful change in the main empirical findings. We also use a narrower definition for entrepreneurs by defining a household as an entrepreneur only when the head of a household earns self-employment income or business income and find that the results are still robust.

in which the head worked in the preceding week, and no one in a household has self-employment income; a non-employed worker is a household of which the head did not work in the preceding week, and no one in a household has self-employment income.

In addition to the CPS, we also use the PSID to check robustness for the income distributions and the transition probabilities across occupations. We choose the PSID data from 1983-1989 because those years are approximately in the median period of the entire sample years of the CPS. For the PSID, we define income as the total taxable income of head and wife. We use the following question in the PSID survey to classify entrepreneurs: *Did you (or anyone else in the family there) own a business or have a financial interest in any business enterprise?* Hence, entrepreneurs are defined in the PSID as families in which at least one person in the family answers yes to the preceding question; workers are households of which the head is working for someone else, and no one in the family answers yes to the above question; and non-employed workers are households in which the head is unemployed and looking for work, retired, permanently disabled, keeping house, in school, or working 10 hours or less per week, and no one in the family answers yes to the above question.

2.2 Entrepreneurs in Income Distribution

To analyze the importance of entrepreneurship over the income distribution, we first examine how entrepreneurs are distributed across the income distribution.

Figure 1 shows the *within-group income share* of entrepreneurs across the income distribution, which is defined as the share of entrepreneurs' income within each of the income groups. The importance of entrepreneurs, in terms of the income share, increases in the higher income groups, especially in the top five percent income group. According to the CPS, entrepreneurs account for 26.7 percent of the total income of the 95th to 100th percentile income group, and 14.8 and 16.6 percent of the total income of the fourth quintile and the 80th to 95th percentile group, respectively. The PSID survey shows a stronger pattern than the CPS, in which entrepreneurial income in the top five percent takes a significantly larger portion of the total income of the group than those of the other income groups.[8]

The within-group income share for entrepreneurs in the top five percent is relatively large because i) the population share of entrepreneurs within the top income group is relatively large and/or ii) entrepreneurs at the top earn relatively high income. Hence, it is also instructive to

[8] Cagetti and De Nardi (2006) also find that entrepreneurs tend to take a large fraction of the total net wealth and a relatively small fraction of the total population from the Survey of Consumer Finances (SCF), regardless of their various definitions. Quadrini (2000) finds the similar tendency in the PSID.

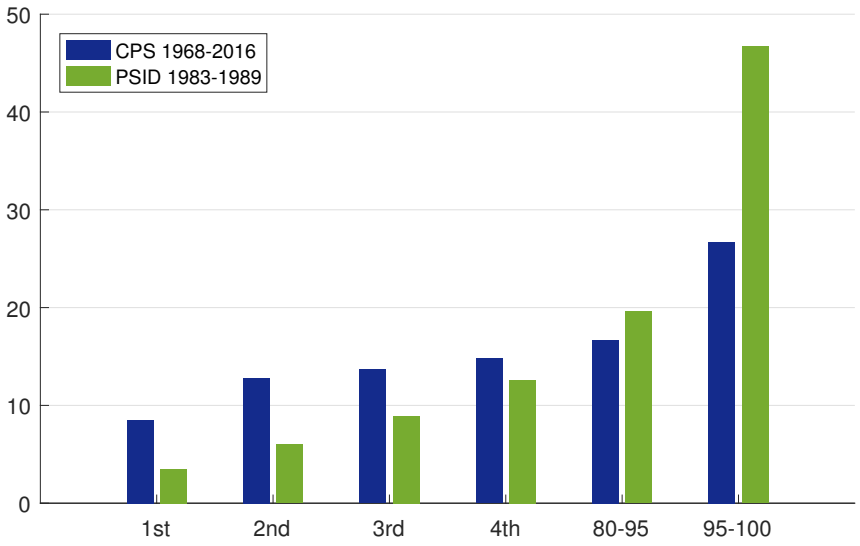

Figure 1: Within-group Income Share of Entrepreneur across Income Groups

Note: The graph shows the average income shares of entrepreneurs within each of the income groups from the CPS 1968-2016 and the PSID 1983-1989. The 1st, 2nd, 3rd, and 4th indicate each of the income quintiles, respectively, and 80-95 and 95-100 denote the income groups of the individuals with income from the 80th to 95th percentiles, and from the 95th to 100th percentiles, respectively.

decompose the within-group income share into two components: the within-group population share and the relative average income.[9] Figure 2 shows the *within-group population share* of entrepreneurs across the income groups, which is defined as the share of entrepreneurs in the total population within each of the income groups. Similar to the within-group income shares for entrepreneurs, the within-group population share of entrepreneurs in the top five percent is highest among all income groups. In the CPS (the PSID), the population fraction of entrepreneurs is 25.0 (40.7) percent in the top five percent income group, while the corresponding numbers in the fourth quintile and the 80th to 95th percentile group are 14.7 (12.4) and 16.5 (19.0) percents, respectively.

The within-group income share of entrepreneurs is larger than the within-group population share of entrepreneurs in the top five percent, while the within-group income shares for entrepreneurs are similar to the within-group population shares of entrepreneurs in the fourth quintile and the 80th to 95th percentile group. This implies that the average income of entrepreneurs is larger than that of other occupations in the top five percent, while the average incomes of entrepreneurs in the fourth quintile and the 80th to 95th percentile group are almost equivalent to those of workers in each of the income groups. Indeed, the relative average income of entrepreneurs to workers in the top five percent is 1.30 in the PSID and 1.10 in the CPS, while the average incomes of entrepreneurs in the fourth quintile and the 80th to95th percentile group are about the same as those of workers in the

[9]We think of the within-group population share of entrepreneurs as an extensive margin, and the relative average income of entrepreneurs as an intensive margin, for the within-group income share of entrepreneurs.

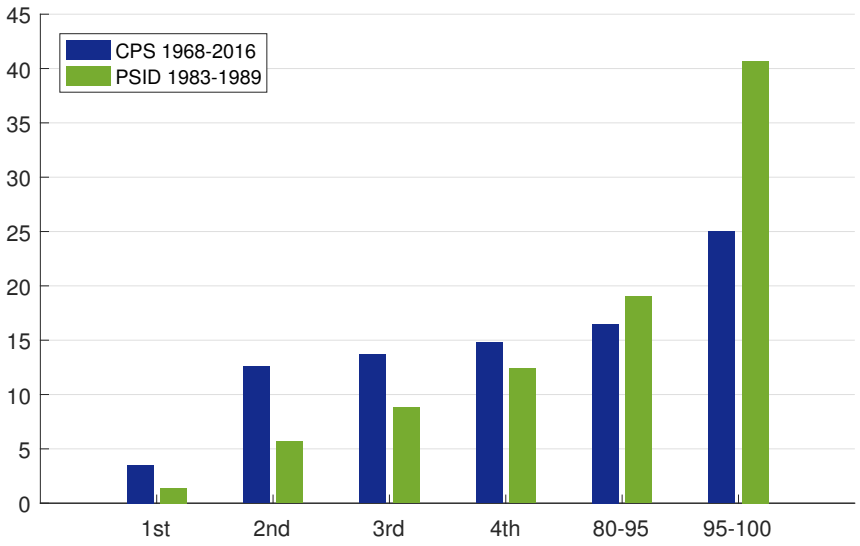

Figure 2: Within-group Population Share of Entrepreneur across Income Groups
Note: The graph shows the average population shares of entrepreneurs within each of the income groups from the CPS 1968-2016 and the PSID 1983-1989.

two data sets (see the bottom panel of Table 6).

In summary, the within-group income share of entrepreneurs in the top five percent is much higher than those in the fourth quintile and the 80th to95th percentile group, since there are more entrepreneurs among top income earners, and entrepreneurs also earn significantly higher income on average than workers, especially in the top five percent. From these empirical findings, it is worth considering entrepreneurial activities to investigate the cyclical behavior of the income shares for the top income group.

2.3 Entrepreneurship and Income Distribution Dynamics

We next investigate the cyclical behaviors of the income distribution, focusing on the role of entrepreneurial activities over the business cycles.[10] The U.S. economy shows a contrasting difference in cyclical properties between low- and high-income groups. The first row of the upper panel in Table 1 reports the contemporaneous correlations of the income shares with output for the first four quintiles, next 15 percent (the 80th to 95th percentile group), and top five percent in the CPS data from 1968 to 2016.[11] All variables are detrended by Hodrick-Prescott filtering before cross correlations are calculated. As already found in Castañeda, Díaz-Giménez, and Ríos-Rull (1998), the income shares of the three lowest quintiles tend to be procyclical, while those of the fourth quintile

[10] Guvenen, Ozkan, and Song (2014) also study the nature of business cycle variation in individual earnings risk for top earners. Relative to Guvenen, Ozkan, and Song (2014), this paper focuses on the cyclical behavior of the income share of the top income group.

[11] Output is defined as private GDP, i.e., GDP minus government expenditure. See Appendix for details.

Table 1: Contemporaneous Correlation of Income Share with Output

	Quintiles				Percentile (%)	
	1st	2nd	3rd	4th	80-95	95-100
CPS 1968-2016						
Total Sample	0.52	0.46	0.41	-0.25	-0.48	-0.05
Excluding Entrepreneur	0.54	0.48	0.23	-0.35	-0.42	-0.32
CPS 1968-2006 (Heathcote, Perri, and Violante, 2010)						
Total Sample	0.64	0.46	0.04	-0.31	-0.49	-0.02
Excluding Entrepreneur	0.64	0.30	0.03	-0.35	-0.47	-0.35

Note: The correlations are calculated after all the variables are detrended with the HP-filter. "Excluding entrepreneur" indicates that we calculate the correlations of the income shares of the income groups with output after excluding entrepreneurs from each of the income groups based on the total sample. The lower panel shows the correlations calculated from the sample (the CPS 1968-2006) of Heathcote, Perri, and Violante (2010), who adopt a Pareto distribution to deal with the top-coding problem.

and the 80th to 95th percentile group are countercyclical. The contemporaneous correlations of the income shares in the three lowest quintiles are 0.52, 0.46, and 0.41, respectively, while those in the fourth quintile and the 80th to 95th percentile group are -0.25 and -0.48, respectively.[12] Therefore, we can easily conclude that, in a relative sense, business expansions benefit low-income earners but hurt high-income earners, resulting in an improvement in inequality in general.[13] As Castañeda, Díaz-Giménez, and Ríos-Rull (1998) find, the transition between employment and non-employment may explain the overall cyclicality of income shares over income groups: relatively more unemployed households in the lower income quintiles are successful in finding jobs during booms, which makes their income shares increase, while rich households lose their shares of the total income since they are already almost fully employed.

The top five percent group, however, does not show any cyclicality with output showing the contemporaneous correlation with output fluctuations close to zero (-0.05), which is a very different behavior from the fourth quintile and the 80-95 percentile. This finding is consistent with those in Castañeda, Díaz-Giménez, and Ríos-Rull (1998) and Parker and Vissing-Jorgensen (2010). To investigate the importance of entrepreneurial activities in the income distribution dynamics, we separate entrepreneurs from the original sample, keeping the income groups unchanged. The second row of the upper panel of Table 1 indicates the cross correlations of the income shares of the six income groups with output when excluding entrepreneurs from the original sample. Interestingly,

[12] It should be noted that average income for each income group is procyclical even if the income share of each income group shows significantly different behaviors over the business cycles: the correlations of detrended average income for each income group with detrended output are between 0.4 and 0.5.

[13] As observed in many studies, income distribution tends to improve during expansions and worsen during recessions. We find that, when output increases, the Gini coefficient declines with a two quarter lag, making $\rho(Y_t, Gini_{t+2}) = -0.54$.

when we drop entrepreneurs from our sample, the correlation between the income share of the top five percent and output is higher in absolute terms and becomes strongly negative, -0.32, while most of the other groups' correlations remain almost unchanged. This implies that entrepreneurs may play a crucial role in reducing the cyclicality of the income share of the top five percent group.[14] As a robustness check, we also use the CPS data constructed by Heathcote, Perri, and Violante (2010), who deal with top-coded observations in a different way from ours.[15] As in the bottom panel of Table 1, the similar pattern of cyclical properties for the income shares is consistently found.

To better understand the role of entrepreneurship in the cyclical property of the income shares, we consider three types of correlations: correlations with output of income shares, population shares, and relative average income across occupations and income groups. In fact, the income share of the occupation i in the income group j is the multiplication of the population share and the relative average income of the occupation i in the income group j. We think that the correlations of income shares may be analyzed by the correlations of the two variables (population shares and relative average income) with output, even though the former correlation is not exactly decomposed into the latter two correlations.[16]

Table 2 summarizes the contemporaneous correlations of income shares with output of each occupation across the six income groups and the correlations with output of the two variables: population shares and relative average income.[17] According to the upper panel in Table 2, as expected, the correlations of the income shares for workers with output are strongly positive in the first three income groups but become negative in the 80th to 95th percentile group and the top five percent income groups. This means that the income shares of workers in the lower income quintiles increase, but the rich workers lose their shares during the boom period. Most of the income shares of entrepreneurs are positively correlated with output, and this behavior particularly contrasts with that of workers in the top five percent income group.[18] The income share of workers in the top five

[14]The acyclical behavior of the income share is also maintained for the top one percent as well: the correlation of the income share of the top one percent with output is -0.11 in our sample, and it is -0.05 in the sample of Heathcote, Perri, and Violante (2010).

[15]Heathcote, Perri, and Violante (2010) assume that each component of income follows Pareto distribution. The sample periods for Heathcote, Perri, and Violante (2010) include survey years 1968 to 2006.

[16]Note that the "income share" or the "population share" for an occupation i in an income group j is different from the "within-group income share" or the "within-group population share" discussed above for an occupation i in an income group j. The former means the portion of the income earned or the population occupied by the households with the occupation i in the income group j out of the "total income or population of the aggregate economy," while the latter means the occupation i's income or population share in the "total income or total population of the income group j." Therefore, the sum of the income shares or population shares over all the occupations and all the income groups is one, while the sum of the within-group income or within-group population shares over all the occupations in the income group j is one.

[17]The relative average income is defined as the ratio of the average income of the occupation in the income group to the average income of the aggregate economy.

[18]The income shares of non-employed workers in most of the income groups show a negative correlation with output, while it is puzzling to have a strong positive correlation of the income share with output of non-employed

percent is negatively correlated with output with a correlation coefficient, -0.31, while entrepreneurs in the same income group show a positive correlation with output, 0.32. However, this contrasting behavior is not observed in the fourth quintile and the 80th to 95th percentile group.

The middle and the bottom panels of Table 2 show which factor is more important between the extensive margin of population and the intensive margin of relative average income in explaining the cyclical behavior of income shares across occupations and income groups. For the cyclical behavior of workers' income share, it seems that the two factors are equally important. Similar to the correlations of income shares for workers with output, the correlations of population shares and relative average income of each of the income groups tend to be strongly positive in the low income quintiles and become negative in the high income groups.[19] On the contrary, the extensive margin is more important than the intensive margin in the cyclical behavior of income shares for entrepreneurs. Focusing on the behavior of the income share of entrepreneurs in the top five percent income group, there is a significant increase in the population of entrepreneurs but not in the relative average income share. The effect of the increase in the population of entrepreneurs in the top five percent income group with a correlation coefficient of 0.37 seems to dominate the acyclical behavior of the relative average income with a correlation coefficient of -0.02, which in turn makes a procyclical income share of entrepreneurs. Interestingly, the positive correlation of the population share for entrepreneurs in the top five percent contrasts with that of workers in the same income group: the population share of entrepreneurs in the top five percent is positively correlated with output (0.37), while workers in the same income group show a negative correlation with output (-0.33). This implies that more entrepreneurial opportunities during expansions induce more households to become entrepreneurs at the top, offsetting a decline in the workers' income share of in the top income group, which results in the acyclical behavior of the income share of the top income earners.

To sum up, the income shares of poor households increase during expansions since there is an active movement from non-employment to employment, but high-income workers have no choice but to lose their shares out of the total income. However, there are also more entrepreneurial opportunities in booms, especially for the top income earners. Accordingly, more entrepreneurial opportunities for the top income earners offset the decline in the income share of the top income earners from the workers' side. As such, we think that entrepreneurial activities play an important role in explaining the cyclical behavior of the income shares over the business cycle, and incorpo-

workers in the first quintile.

[19] Another finding is that some non-employed households start a business out of necessity during booms. For example, according to the middle panel of Table 2, in the first income quintile, the correlation of the population share of non-employed households with output is negative, while that of entrepreneurs is positive. This implies that, during expansions, some non-employed households at the bottom of the income distribution start a business.

Table 2: Decomposition of Cyclical Property of Income Share

	Quintiles				Percentile (%)	
	1st	2nd	3rd	4th	80-95	95-100
Correlation of Income Share with Output						
Worker	0.49	0.69	0.65	0.10	-0.29	-0.31
Entrepreneur	0.26	0.22	0.33	0.15	0.02	0.32
Non-employed Worker	0.55	-0.48	-0.59	-0.55	-0.39	-0.10
Correlation of Population Share with Output						
Worker	0.60	0.78	0.56	0.36	0.17	-0.33
Entrepreneur	0.25	-0.14	0.25	0.20	0.13	0.37
Non-employed Worker	-0.53	-0.77	-0.65	-0.55	-0.34	-0.12
Correlation of Relative Average Income with Output						
Worker	0.51	0.46	0.34	-0.33	-0.51	-0.12
Entrepreneur	0.45	0.59	0.39	-0.26	-0.44	-0.02
Non-employed Worker	0.54	0.40	0.39	-0.02	-0.42	-0.03

Note: "Income share," "population share," and "relative average income" are defined as the income share of each occupation in each income group out of the total aggregate income, the population share of each occupation in each income group out of the total population, and the ratio of the average income of each occupation in each of the income group to the aggregate average income, respectively. Therefore, the sum of all the income shares or all the population shares over both the occupations and the income groups is one.

rating entrepreneurs in a model may be critical in replicating the acyclical behavior of the income shares of the top income earners. Therefore, we will explicitly introduce entrepreneurial activities into the model and investigate their importance in explaining the cyclical behavior of the income distribution.

3 The Model

We build a simple dynamic stochastic general equilibrium model with a large population of heterogeneous households in labor efficiency with an incomplete capital market, and add additional features of an entrepreneurial opportunity with heterogeneous entrepreneurial productivity shocks.

The key features of our model economy are summarized as follows. First, as in Quadrini (2000), Cagetti and De Nardi (2006), and Terajima (2006), our model also develops entrepreneurship, by introducing an occupational decision for a household to become an entrepreneur. A household makes an occupational decision among the three occupational choices: employment, entrepreneurship, and no work. A household may decide to supply labor to the market as a *worker*, to participate in their own business with labor and capital as an *entrepreneur*, or not to work due to unfavorable

individual labor productivity shocks and/or low entrepreneurial productivity shocks, which we call a *non-employed worker*. Second, we introduce aggregate productivity shocks in an occupational choice model, while most of the previous studies on entrepreneurship only focus on the steady state properties, not on the business cycle properties. By introducing aggregate productivity shocks to an occupational choice model, we can examine the importance of entrepreneurs' behavior in business cycle properties of income shares for the top income earners.

3.1 Preferences

A household maximizes the expected lifetime utility over consumption of a market good, c_t, and consumption of a home-produced or non-market good, h_t:

$$\max_{\{c_t,h_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \right], \tag{1}$$

with

$$u(c,h) = \omega \log c + (1-\omega) \log h, \tag{2}$$

where $\mathbb{E}_0[\cdot]$ denotes the expectation operator conditional on the information available at period 0, $0 < \beta < 1$ is the time discount factor, and $0 < \omega < 1$ represents weight of the market consumption.

3.2 Heterogeneity

There are two types of heterogeneity in the model economy: labor efficiency, x, and entrepreneurial productivity, e. A household faces idiosyncratic risks in x and e, and the shocks follow a stochastic process with transition probabilities $Q_s(s'|s) = \Pr(s_{t+1} = s'|s_t = s)$, where $s \in \{x, e\}$. These idiosyncratic shocks are assumed to be independently distributed across the households and identically distributed within the households with the same s. In addition, we assume that the two idiosyncratic shocks, x and e, are uncorrelated with each other and follow AR(1) processes:

$$\ln s' = (1-\rho_s) \ln \overline{s} + \rho_s \ln s + \epsilon_s, \quad \epsilon_s \sim N(0, \sigma_s), \tag{3}$$

where $\overline{s}$ denotes steady state values of $s \in \{x, e\}$. The capital market is incomplete following Huggett (1993) and Aiyagari (1994): the physical capital, a, is the only asset available to insure against idiosyncratic risks arising from x and e.

3.3 Technology

There are two production sectors in the model economy: a corporate sector and an entrepreneurial sector.[20] The corporate production sector employs labor and capital, and pays wages and interest rates to the production factors as in a conventional aggregate production in the DSGE literature. The market wage and rental rates are determined by the supply of and demand for the production factors in the corporate sector. An individual entrepreneur can also employ workers but faces collateral constraints. Therefore, entrepreneurial production activities can be considered self-employment, small business firms hiring production factors, or independent business projects, which is in line with Quadrini (2000). The depreciation rate for physical capital, $0 < \delta < 1$, is common in both of the production sectors.

Corporate Production The production technology in the corporate sector is represented by a constant-returns-to-scale Cobb-Douglas production function:

$$F(K, L, z) = zK^{\alpha}L^{1-\alpha}, \tag{4}$$

where K and L denote capital and effective labor used in the corporate production, respectively, and z is an aggregate productivity shock which follows an AR(1) stochastic process with transition probabilities, $Q_z(z'|z) = Pr(z_{t+1} = z'|z_t = z)$:

$$\ln z' = \rho_z \ln z + \epsilon_z, \quad \varepsilon_z \sim N(0, \sigma_z^2). \tag{5}$$

It is assumed that the aggregate shocks and the two idiosyncratic shocks for each individual are independent of one another.

Entrepreneurial Production Each firm in the entrepreneurial sector run by an entrepreneur with ability e, who uses capital k and effective outside labor l, produces output via a decreasing-returns-to-scale (DRS) technology:

$$f(k, l, z, e) = ze\left(k^{\alpha}l^{1-\alpha}\right)^{v}, \tag{6}$$

[20] This modelling choice of having a corporate sector in addition to the entrepreneurs is conventional in the literature on entrepreneurship (e.g., Quadrini (2000) and Cagetti and De Nardi (2006)).

where $0 < v < 1$ denotes a span-of-control parameter.[21] Note that an entrepreneur is required to hire outside labor and earn profits after wages and rents are paid. An entrepreneur faces collateral constraints,

$$k \leq \theta a, \tag{7}$$

where $\theta \geq 1$. θ captures the degree of capital market imperfection for entrepreneurial production activities: capital market is perfect when $\theta = \infty$, while it is completely shut down when $\theta = 1$. Given the entrepreneurial production technology in a DRS form, the collateral constraints are not always binding.[22]

Home Production In the economy, there is a home production technology which each household has access to. Based on this technology, households can produce fixed units of the home-produced good in any given period. Importantly, it is assumed that home production technologies are different depending on households' occupational statuses. Specifically, each occupation produces different units of the non-market good, and the home production for a worker, an entrepreneur, or a non-employed worker is denoted by $\overline{h}_j > 0$, where $j = W$, E, or N, respectively.[23] According to Hurst and Pugsley (2011) and Yurdagul (2017), households may want to be entrepreneurs due to non-pecuniary reasons – for example, "wanting flexibility over schedule" or "to be one's own boss." In this sense, the different home production technologies depending on occupational statuses may be considered to reflect the non-pecuniary motives to some extent.

3.4 Household's Problem

Worker's Problem The budget constraint of a worker is:

$$c = wx + (1 + r)a - a', \tag{8}$$

and

$$c \geq 0 \text{ and } a' \geq \underline{a}, \tag{9}$$

[21] Many existing papers in the entrepreneurship literature employ a decreasing returns technology for entrepreneurs to capture the limited span-of-control. See, for instance, Buera, Kaboski, and Shin (2011), Buera and Shin (2013), and Midrigan and Xu (2014).

[22] In case that the entrepreneurial production follows a constant-returns-to-scale (CRS) technology (i.e., $v = 1$), the collateral constraints are always binding, and the factor demand functions will be linear in asset holdings.

[23] We assume that $\overline{h}_N > \overline{h}_W$ and $\overline{h}_N > \overline{h}_E$. The positive units of the home-produced good for workers or entrepreneurs ($\overline{h}_E, \overline{h}_E > 0$) are needed for reasonable distributions of workers and entrepreneurs across the income distribution.

where w and r are the wage rate and interest rates on assets a, respectively. We impose a restriction that workers face a borrowing constraint: the level of asset holdings, a', cannot be below $\underline{a}$, implying that there is a lower limit for the level of liability. The state variables for a household are the vector $\Omega \equiv (a, x, e)$ and the economy-wide state is described by a distribution and an aggregate shock, (μ, z), where μ is a joint distribution of assets, idiosyncratic labor productivity shocks, and entrepreneurial productivity shocks. The value function for a worker, denoted by $V_W(\Omega; \mu, z)$, is

$$V_W(\Omega; \mu, z) = \max_{c,a'} \{u(c, \overline{h}_W) + \beta \mathbb{E}\left[V(\Omega'; \mu', z')\right]\} \tag{10}$$

subject to

$$c = wx + (1 + r)a - a', \ c \geq 0 \text{ and } a' \geq \underline{a}, \tag{11}$$

and

$$\mu' = \phi(\mu, z), \tag{12}$$

where ϕ denotes a transition operator for the distribution, μ.

Entrepreneur's Problem If a household decides to be an entrepreneur, its value function $V_E(\Omega; \mu, z)$ is:

$$V_E(\Omega; \mu, z) = \max_{c,a',k,l} \{u(c, \overline{h}_E) + \beta \mathbb{E}\left[V(\Omega'; \mu', z')\right]\}, \tag{13}$$

subject to

$$c = f(k, l, z, e) - (r + \delta)k - wl + (1 + r)a - a', \ c \geq 0, \ a' \geq \underline{a}, \tag{14}$$

$$k \leq \theta a, \ k > 0, \text{ and } \mu' = \phi(\mu, z). \tag{15}$$

There are two notable features in the entrepreneur's problem. First, entrepreneurs do not pay wages to themselves: there are no wage earnings in the budget constraint of entrepreneurs in Equation (14). Effective outside labor used for the entrepreneurial firm, l ,indicates only outside labor but not the entrepreneur's own labor while capital input, k, includes a part or all of the entrepreneur's own capital, a. Entrepreneurs earn interests for supplying physical capital, a, allocated in the corporate sector, and profits for supplying time efforts as a manager. Second, the

constraint $k > 0$ and the borrowing constraint $k \leq \theta a$ imply that a household should have positive net assets to be an entrepreneur.

Non-employed Worker's Problem The value function for a non-employed worker who decides not to work, denoted by $V_N(\Omega; \mu, z)$, is:

$$V_N(\Omega; \mu, z) = \max_{c,a'} \{u(c, \overline{h}_N) + \beta \mathbb{E}\left[V(\Omega'; \mu', z')\right]\}, \tag{16}$$

subject to

$$c = (1 + r)a - a',\ c \geq 0,\ a' \geq \underline{a},\ \text{and}\ \mu' = \phi(\mu, z). \tag{17}$$

Household's Occupational Decision A household's value function $V(\Omega; \mu, z)$ is defined as:

$$V(\Omega; \mu, z) = \max\left\{V_W(\Omega; \mu, z),\ V_E(\Omega; \mu, z),\ V_N(\Omega; \mu, z)\right\}, \tag{18}$$

and its occupational decision is a worker, an entrepreneur, or a non-employed worker when $V(\Omega; \mu, z) = V_j(\Omega; \mu, z)$, $j = W$, E, or N, respectively.

3.5 Definition of Equilibrium

A recursive competitive equilibrium consists of a set of input functions in the corporate production sector $\{K(\mu, z),\ L(\mu, z)\}$, pricing functions $\{r(\mu, z),\ w(\mu, z)\}$, optimal decision rules $\{c(\Omega; \mu, z),\ a'(\Omega; \mu, z),\ h(\Omega; \mu, z),\ k(\Omega; \mu, z),\ l(\Omega; \mu, z)\}$, value functions $\{V(\Omega; \mu, z),\ V_W(\Omega; \mu, z),\ V_E(\Omega; \mu, z),\ V_N(\Omega; \mu, z)\}$, and a law of motion for the distribution $\mu' = \phi(\mu, z)$, such that:

1. A household's optimization: The optimal decision rules $c(\Omega; \mu, z)$, $a'(\Omega; \mu, z)$, $h(\Omega; \mu, z)$, $k(\Omega; \mu, z)$, and $l(\Omega; \mu, z)$ solve the value functions $V(\Omega; \mu, z)$, $V_W(\Omega; \mu, z)$, $V_E(\Omega; \mu, z)$, and $V_N(\Omega; \mu, z)$, given $r(\mu, z)$, $w(\mu, z)$, and $\phi(\mu, z)$.[24]

2. The corporate firm's optimization: For all μ and z,

$$r(\mu, z) = F_K(K(\mu, z), L(\mu, z), z) - \delta,$$

$$w(\mu, z) = F_L(K(\mu, z), L(\mu, z), z).$$

3. Market clearing: For all μ and z,

[24] It should be noted that $k(\Omega; \mu, z) = 0$ and $l(\Omega; \mu, z) = 0$ for non-entrepreneurs.

$$L(\mu, z) + \int l(\Omega; \mu, z) d\mu = \int x d\mu_W,$$[25]

$$K(\mu, z) + \int k(\Omega; \mu, z) d\mu = \int a d\mu,$$

where μ_W is the measure for workers.[26]

4. Consistency of individual and aggregate behavior: The law of motion for the distribution $\phi(\mu, z)$ is consistent with that implied by the optimal decision rule $a'(\Omega; \mu, z)$.

3.6 Calibration

Table 3 summarizes the parameter values. As is standard in the business cycle literature, we use the conventional parameter values adopted from the various previous studies. To the extent possible, we calibrate our model's baseline to 1968-2016.

As for individual labor productivity shocks, many studies attempt to estimate the parameters, ρ_x and σ_x (Floden and Lindé, 2001; French, 2005; Chang, Kim, and Schorfheide, 2013; Takahashi, 2015). It is common that individual labor productivity shocks have a large variance and a high persistence. We use $\rho_x = 0.939$ and $\sigma_x = 0.287$ following Chang, Kim, and Schorfheide (2013), who estimate them with the PSID. The unconditional expected value of individual productivity shocks, $\mathbb{E}(x)$, is normalized at one. The borrowing limit, $\underline{a}$, is set to -2.0.[27]

Empirical evidence for parameters of individual entrepreneurial productivity shocks can be found in the firm dynamics literature (e.g., Cooper and Haltiwanger (2006), Foster, Haltiwanger, and Syverson (2008), Castro, Clementi, and Lee (2015), and Lee and Mukoyama (2015)). Many empirical studies in this literature find that firm-level productivity processes are strongly persistent: the persistence estimate in a quarterly value ranges from 0.837 to 0.995. Hence, we simply assume that the persistence parameter of entrepreneurial shocks is the same as that of labor productivity shocks: $\rho_e = \rho_x = 0.939$. This parameter value is similar to the estimate of Foster, Haltiwanger, and Syverson (2008) and a bit less than the one estimated by Castro, Clementi, and Lee (2015).[28]

[25] More precisely, $\mu_W(a, x, e) = \begin{cases} \mu(a, x, e) & \text{if a household is a worker} \\ 0 & \text{otherwise} \end{cases}$.

[26] As mentioned above, since the entrepreneur's time effort is only used for the entrepreneurial activity, their labor productivity is not included in $l(\Omega; \mu, z)$, and the total effective labor in the market is only provided by workers.

[27] The value of $\underline{a}(= -2)$ implies that the maximum debt is around twice the quarterly average income of a household in the model.

[28] Foster, Haltiwanger, and Syverson (2008) investigate the nature of selection and productivity growth using industry-level data and find that the average estimate of the persistent parameter is 0.94. Castro, Clementi, and Lee (2015) estimate the volatility and persistence parameters of plant-level idiosyncratic shocks in U.S. manufacturing and find that the average estimate of the persistent parameter for sales is 0.88. These numbers are the quarterly values converted from their annual estimates.

Table 3: Parameters of the Model Economy

Parameter	Value	Description	Source or Target Moments
From Data or Previous Literature			
ρ_x	0.939	Persistence of x shocks	Chang, Kim, and Schorfheide (2013)
σ_x	0.287	Standard deviation of x shocks	Chang, Kim, and Schorfheide (2013)
$\mathbb{E}(x)$	1.0	Unconditional mean of x shocks	Normalized
$\underline{a}$	-2.0	Borrowing constraint	See text
ρ_e	0.939	Persistence of e shocks	See text
v	0.88	Parameter for DRS	Cagetti and De Nardi (2006)
θ	1.5	Entrepreneur's collateral constraint	Evans and Jovanovic (1989)
α	0.36	Capital income share	Standard
ρ_z	0.95	Persistency of aggregate shocks	Kydland and Prescott (1982)
σ_z	0.007	Standard deviation of aggregate shocks	Kydland and Prescott (1982)
δ	0.025	Depreciation rate	Standard
$\overline{h}_N$	0.500	Home production for non-emp. workers	See text
Calibrated to Target Steady State Values			
β	0.97125	Discount factor	Real interest rates
$\overline{h}_W$	0.204	Home production for workers	Population share of workers
$\overline{h}_E$	0.365	Home production for entrepreneurs	Population share of entrepreneurs
$\mathbb{E}(e)$	1.385	Unconditional mean of e shocks	Entrepreneurs' income share
σ_e	0.385	Standard deviation of e shocks	Gini index for entrepreneurs' income

For the standard deviation of entrepreneurial productivity shocks, we set $\sigma_e = 0.385$ to match the income Gini coefficient for entrepreneurs, 0.55 in the data. Given these calibrated parameter values, the stochastic process of entrepreneurial productivity is riskier than the one of workers, i.e., $\sigma_e > \sigma_x$.

The unconditional expected value of idiosyncratic productivity shocks $\mathbb{E}(e)$ is chosen to target the income share of entrepreneurs, which is around 20 percent in the data.[29] The borrowing constraint parameter for entrepreneurial capital, θ, is set to 1.5 following the empirical results of Evans and Jovanovic (1989), implying that entrepreneurs can borrow up to 50 percent of their own assets to invest in their business in addition to their own capital. The degree of decreasing returns to scale for the entrepreneurial production, v, is set to 0.88 following Abraham and White (2006) and Cagetti and De Nardi (2006). For aggregate productivity shocks, we simply follow Kydland and Prescott (1982) and choose $\rho_z = 0.95$ and $\sigma_z = 0.007$.[30] The capital income share α is 0.36,

[29] This calibration is consistent with that of Quadrini (2000) and Cagetti and De Nardi (2006).

[30] Regarding aggregate productivity shocks, alternative parameterizations in a reasonable range do not produce any qualitative difference from the benchmark model in terms of correlations with output and volatilities relative to output.

and the depreciation rate δ is 2.5 percent per quarter.

There are four more parameters to be calibrated: β, $\overline{h}_W$, $\overline{h}_E$, and $\overline{h}_N$.[31] We choose β to be consistent with a one percent quarterly return to capital, and $\overline{h}_W$ and $\overline{h}_E$ are set to target the population shares of workers and entrepreneurs, respectively. We choose the home-produced good for a non-employed worker, $\overline{h}_N$, so that the total home production, $\int h(\Omega)d\mu$, is to match the 30 percent of GDP in the steady state, which is consistent with the empirical findings of Giannelli, Mangiavacchi, and Piccoli (2012) and Vargha, Gal, and Crosby-Nagy (2017).[32]

4 Findings

4.1 Properties of the Steady State

4.1.1 Key Aggregate Moments of Distributions

In this subsection, we present the aggregate moments of distributions in the benchmark model economy. We summarize the data and the model counterparts of the targeted (the upper panel) and the untargeted moments (the bottom panel) in Table 4.

We are successful in targeting four aggregate moments of distribution: the population share of workers, the population share of entrepreneurs, the Gini coefficient for entrepreneurs' income,[33] and the income share of entrepreneurs. Our model economy also fits the untargeted moments reasonably well. The income and wealth Gini coefficients are well reproduced, and the income Gini coefficient for workers is also close to the data.[34] The wealth share of entrepreneurs is 26 percent in the benchmark model, which is a bit less than that in the U.S. data (35 percent). The fractions of staying entrepreneurs and workers in the model economy are also broadly consistent with the data. In the PSID, 79 (90) percent of entrepreneurs (workers) in the current year remain as entrepreneurs (workers) in the following year, and the benchmark model produces a reasonable number of 61 (95) percent.

[31] Notice that, since a home-produced good for each occupation is assumed to be constant over time, any choice of ω in Equation (2) does not affect quantitative results.

[32] Giannelli, Mangiavacchi, and Piccoli (2012) estimate the gross value of total home production between 12 and 47 percent of GDP in 24 European countries, and Vargha, Gal, and Crosby-Nagy (2017) also find that the gross values of home production are between 19 and 46 percent of national GDPs in 14 European countries

[33] The wealth Gini index for entrepreneurs in the model economy is less than what we observe in the data: the wealth Gini coefficient for entrepreneurs is 0.53 in the model, while it is 0.76 in the U.S. data. This is because entrepreneurs should have positive net wealth in the model, while some entrepreneurs hold negative net assets in the data (around 4 percent in the PSID).

[34] The model economy broadly matches the wealth Gini index for workers in the U.S data: the Gini index for workers' wealth is 0.84 in the model, while it is 0.74 in the data.

Table 4: Summary of Aggregate Moments of Distribution

Moment	Data	Model
Targeted		
Population share of worker	0.60	0.60
Population share of entrepreneur	0.11	0.11
Gini coefficient for entrepreneurs' income	0.55	0.55
Income share of entrepreneur	0.20	0.21
Untargeted		
Gini coefficient for income	0.53	0.57
Gini coefficient for wealth	0.80	0.80
Gini coefficient for workers' income	0.50	0.47
Wealth share of entrepreneur	0.35	0.26
Fraction of staying entrepreneur	0.79	0.61
Fraction of staying worker	0.90	0.95

Note: The wealth share of entrepreneurs and the Gini coefficient for wealth are from the PSID 1989, and the fraction of staying entrepreneurs (workers) is the average percentage of entrepreneurs (workers) in the following year out of the entrepreneurs (workers) in the current year over the period from the PSID 1983-1989. The remaining statistics are the averages from the PSID 1983-1989 or the CPS 1968-2016.

4.1.2 Wealth and Income Distributions

We next compare wealth and income distributions in the U.S. data and in the benchmark model economy. As for the steady state income distribution shown in the left panel of Figure 3, the model economy replicates the income distribution of the U.S. reasonably well. The implied income Gini coefficient based on the Lorenz curve of the simulated model is 0.57, which is similar to the Gini coefficient of the data, 0.53. The benchmark model with entrepreneurs performs much better in the wealth distribution than the model without entrepreneurs, where households face only individual labor productivity shocks, and there are only two types of occupations (employment and non-employment).[35] As shown in the right panel of Figure 3, the inequality of wealth distribution in the benchmark model economy is much better matched with the data than in the model without entrepreneurs: the benchmark model economy makes the wealth Gini index about 0.80, which is the same as the U.S. data, while the corresponding value for the model without entrepreneurs is only 0.65. We think that the introduction of entrepreneurial opportunities in the benchmark model is the main factor for successfully generating the observed unequal wealth distribution, which is in line with Quadrini (2000) and Cagetti and De Nardi (2006).[36]

[35] See Section 4.3.2 for detailed discussions about the model without entrepreneurs and calibration strategies for this counterfactual economy.

[36] We do not think that the wealth distribution itself plays an important role in replicating the cyclical behavior of income distribution as found in Castañeda, Díaz-Giménez, and Ríos-Rull (1998) but think that the cyclicality of the

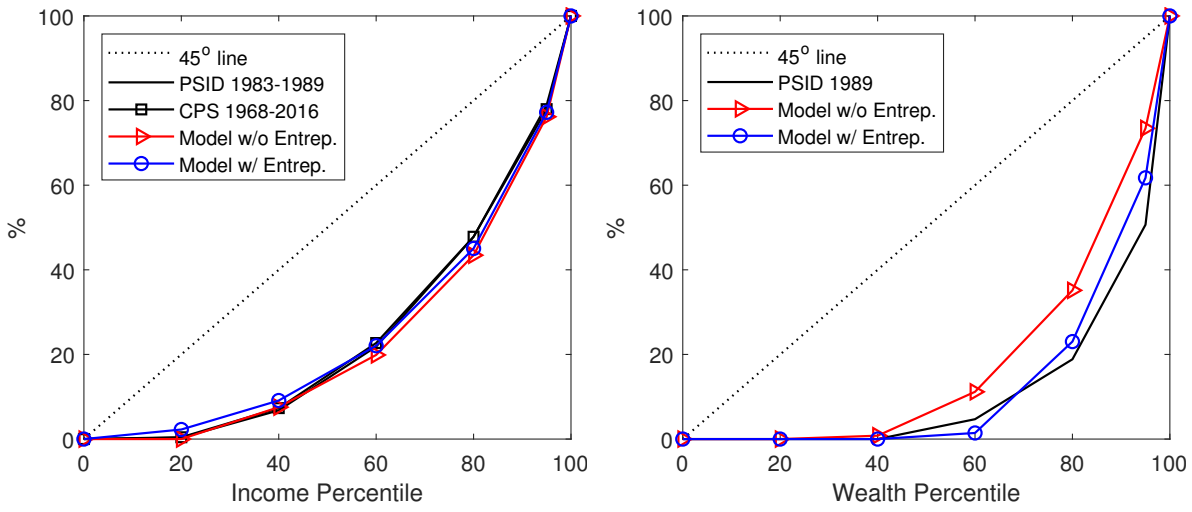

Figure 3: Lorenz Curves for Income and Wealth

Note: The vertical axis shows the cumulative share of wealth or income, and the horizontal axis represents the cumulative percentages of populations ranked by wealth or income. Lorenz curves of income (left panel) and wealth (right panel) are drawn by using the cumulative percentage shares only at seven points of percentiles of the U.S. data and the model economies: 0, 20, 40, 60, 80, 95, and 100 percentile.

Another interesting finding of this study is the role of entrepreneurship in wealth mobility. Table 5 reports the five-year wealth transition probability matrices among each of the occupational statuses in the benchmark model. Quadrini (2000) empirically documents that entrepreneurs experience a higher upward mobility in wealth distribution than non-entrepreneurs. This empirical pattern is well reproduced by the benchmark model economy in that it reasonably replicates the substantial differences in the mobility patterns of entrepreneurs and non-entrepreneurs. In the model economy, entrepreneurs tend to stay in or move to higher positions, while non-entrepreneurs tend to stay in or move to lower positions of wealth. For example, the percentage of households that move up to upper wealth groups is higher for staying entrepreneurs than for switching entrepreneurs, while the percentage of households that move up to upper wealth groups is larger for switching non-entrepreneurs than for staying non-entrepreneurs.

4.1.3 Occupation Choices

It is also instructive to see how households decide their occupations. We provide a graphical representation for occupational decision rules in Figure 4, which illustrates the occupational decisions depending on labor productivity, entrepreneurial productivity, and asset holdings. The decision rules are computed from the steady-state model economy (the model economy without aggregate

entrepreneurial opportunity itself is crucial to accounting for the income distribution dynamics.

Table 5: Five-year Transition Probabilities for Wealth: Benchmark Model

	Group I	Group II	Group III	Group I	Group II	Group III
	Staying Entrepreneur			Switching Entrepreneur		
Group I	4.4	56.1	39.4	69.9	25.1	5.0
Group II	2.0	44.5	53.5	35.0	45.0	20.0
Group III	0.1	13.1	86.8	2.0	27.8	70.3
	Staying Non-entrepreneur			Switching Non-entrepreneur		
Group I	73.3	20.4	6.4	4.7	63.7	31.6
Group II	40.7	38.3	21.0	2.8	52.5	44.7
Group III	5.4	33.4	61.2	0.4	23.8	75.7

Note: The upper panel reports the transition probabilities across three wealth groups for the households that were entrepreneurs five years ago, and the bottom panel reports the transition probabilities across the wealth groups for the households that were not entrepreneurs five years ago. Group I is the 0-40 percentile, Group II is the 40-70 percentile, and Group III is the 70-100 percentile.

shocks) as a result of optimization.[37] The purple, yellow, and green regions indicate the region where the occupational decision is worker, non-employed worker, and entrepreneur, respectively. For simplicity, only four cases of entrepreneurial shocks are provided to show how the occupational decision changes over entrepreneurial productivity. There are several notable points from the occupational decision. First, with a higher entrepreneurial productivity, the region of a decision as an entrepreneur is larger. Second and interestingly, there are some labor productivity-asset combinations where the region of workers becomes larger as the level of entrepreneurial productivity is higher. For example, when entrepreneurial productivity is 0.9, a household that has labor productivity of around 4 and owns assets of around 30 is non-employed while a household becomes a worker when entrepreneurial productivity is 1.2. This is due to the incentive to accumulate assets for the possibility of becoming an entrepreneur in the future since the entrepreneurial productivity shock is persistent. Third, when entrepreneurial productivity is relatively low, the region of a decision as a non-employed worker is larger with a higher asset level, which is due to wealth effects.

4.2 Entrepreneurship in Income Distribution

To analyze the relationship between entrepreneurship and income inequality focusing on the top income earners, we compare the within-group income shares of each occupation across the income groups generated by the model economy with those of the U.S. data.[38] The upper panel of Table 6 shows the within-group income shares of entrepreneurs across the income distribution in the benchmark model. Overall, the model economy reproduces the pattern of the within-group income

[37] As discussed in Equation (18), the occupational decision depends on asset, labor productivity shock, entrepreneurial shock, economy-wide distribution of assets, and aggregate shocks.

[38] See Appendix for detailed information on the various distributions of each occupation across the income groups.

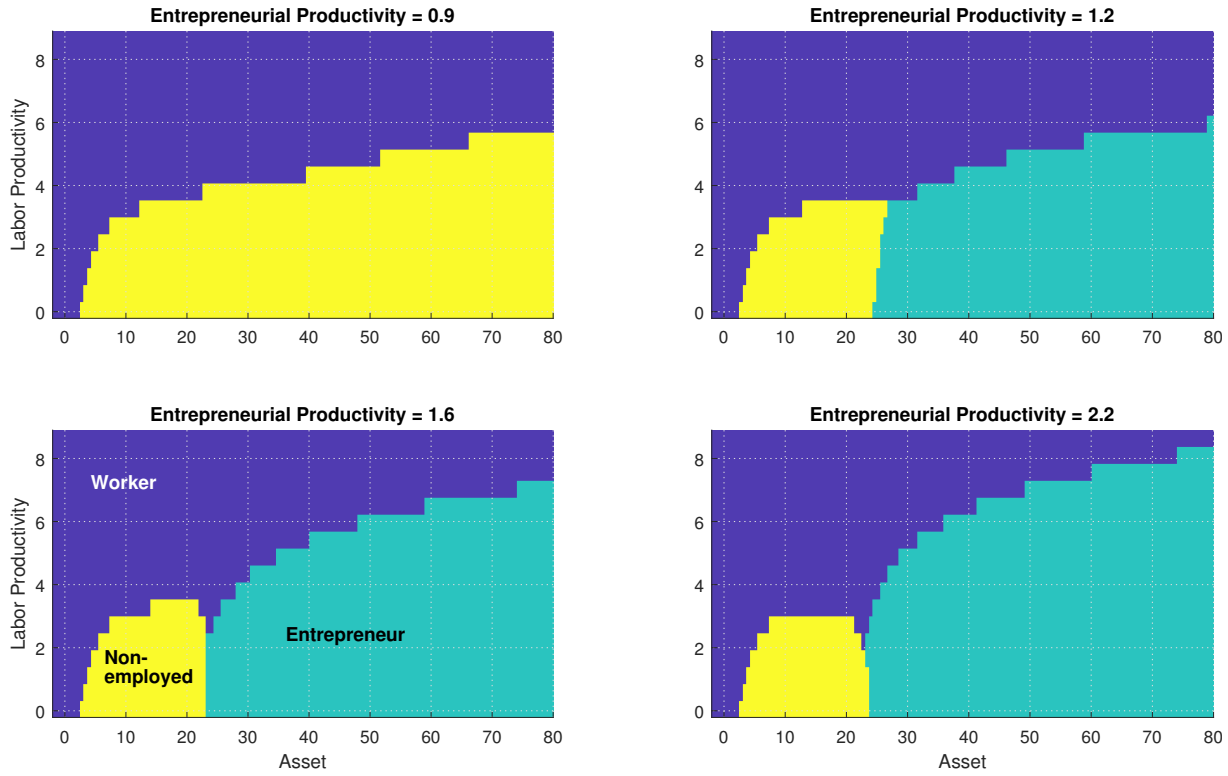

Figure 4: Occupational Decision Rule

Note: The purple, yellow, and green regions illustrate the regions where the occupational decision is worker, non-employed worker, or entrepreneur, respectively, assuming the stationary distribution of assets. The figures are drawn for a part of the whole range of assets of interest and four cases of entrepreneurial shocks.

shares across income groups found in the U.S. data rather well: entrepreneurs are more important in the high-income groups than in the low-income groups. In particular, the benchmark economy excels at producing empirically realistic within-group income shares of the higher income groups. The within-group income shares of entrepreneurs in the fourth quintile and in the 80th to 95th percentile group are 14.8 (12.6) percent and 16.6 (19.6) percent, respectively, in the CPS (the PSID), and the model produces reasonable numbers of 13.8 percent and 16.6 percent, respectively. Consistent with the U.S. data, the within-group income share is much higher in the top five percent income group than in the other income groups in the benchmark economy: the within-group income share of entrepreneurs for the top five percent in the model economy is 39.1 percent, which falls within the range of the two data sets.

The income shares of entrepreneurs in each of the income groups can be divided into two components: one is the within-group population share of entrepreneurs (extensive margin), and the other is the relative average income (intensive margin). The middle panel of Table 6 shows the within-group population shares of entrepreneurs across the income groups for the U.S. data and the benchmark model. Similar to the data, the model economy shows the largest within-group

Table 6: Distribution of Entrepreneur across Income Group

	Quintiles				Percentile (%)		Total
	1st	2nd	3rd	4th	80-95	95-100	
Within-group Income Share of Entrepreneur							
PSID	3.5	6.0	8.9	12.6	19.6	46.7	21.0
CPS	8.5	12.7	13.7	14.8	16.6	26.7	17.5
Benchmark Model	2.7	7.2	12.5	13.8	16.6	39.1	20.8
Within-group Population Share of Entrepreneur							
PSID	1.4	5.7	8.8	12.4	19.0	40.7	10.5
CPS	3.5	12.6	13.7	14.7	16.5	25.0	12.6
Benchmark Model	1.7	6.8	12.2	13.7	16.2	30.6	10.8
Relative Income of Entrepreneur to Worker							
PSID	0.98	0.99	1.01	1.02	1.04	1.30	1.71
CPS	1.06	0.94	1.00	1.00	1.01	1.10	1.09
Benchmark Model	1.10	1.05	1.02	1.01	1.02	1.44	1.71

Note: "Within-group Income Share" means the share of entrepreneurs' income within each of the income groups, "Within-group Population Share" denotes the share of entrepreneurs in the total population within each of the income groups, and "Relative Income to Worker" means the ratio of the average income of the entrepreneurs in the income group to that of the workers in the income group.

population share of entrepreneurs in the top five percent, 30.6 percent, of all the income groups. The corresponding numbers for the fourth quintile and the 80th to 95th percentile group are 13.7 percent and 16.2 percent, respectively, which are very similar to what the data show. As can be seen in the bottom panel of Table 6, the average income of entrepreneurs relative to the average income of workers is higher than one in the top five percent income group in the data (1.30 in the PSID or 1.10 in the CPS), which is appropriately reproduced in the benchmark model as 1.44 even though it is a bit higher than the data. There is no significant difference between the relative average income of entrepreneurs and that of workers in all other income groups in the data and also in the model.

Therefore, the model economy successfully replicates the income shares, population shares, and relative average income of entrepreneurs across the income groups in the data. We think that the significant income share of entrepreneurs in the top five percent income group may play an important role in explaining the business cycle behavior of income distribution, and more detailed discussion follows below.

Table 7: Volatilities and Comovements of Aggregate Variables

	σ_Y	σ_C/σ_Y	σ_I/σ_Y	σ_E/σ_Y	$\sigma_{Y/E}/\sigma_Y$
U.S. Data	2.51	0.49	3.64	0.75	0.60
Benchmark Model	1.39	0.57	3.40	0.26	0.80
	cv_{PE}/cv_{PW}	cv_{IE}/cv_{IW}	σ_{YE}/σ_Y	σ_{YW}/σ_Y	σ_{Gini}
U.S. Data	2.75	4.98	2.08	1.14	0.43
Benchmark Model	2.19	3.07	2.27	1.61	0.29
	$\rho(Y,Y)$	$\rho(C,Y)$	$\rho(I,Y)$	$\rho(E,Y)$	$\rho(\frac{Y}{E},Y)$
U.S. Data	1.00	0.82	0.92	0.80	0.66
Benchmark Model	1.00	0.91	0.95	0.82	0.98
	$\rho(PW,Y)$	$\rho(PE,Y)$	$\rho(IW,Y)$	$\rho(IE,Y)$	$\rho(Gini,Y)$
U.S. Data	0.77	0.23	-0.23	0.24	-0.54
Benchmark Model	0.62	0.14	-0.18	0.17	-0.44

Note: σ_q, cv_q, and $\rho(q,Y)$ are standard deviation of q, coefficient of variation (the ratio of the standard deviation to the mean) of q, and cross correlation of q with output (Y), respectively. C, I, E, PW, PE, IW, IE, YW, YE, and $Gini$ denote consumption, investment, total employees, population share of workers, population share of entrepreneurs, income share of workers, income share of entrepreneurs, total income of workers, total income of entrepreneurs, and Gini coefficients, respectively. $\rho(Gini,Y)$ indicates the two-year lagged correlation of the Gini coefficient with output, $\rho(Gini_{t+2},Y_t)$ for the U.S. data but the contemporaneous correlation, $\rho(Gini_t,Y_t)$ for the benchmark model, since the lagging behavior of the Gini coefficient is not a main issue in this study. All variables other than PW, PE, IW, IE, and $Gini$ are logged and all variables are detrended by the HP filter.

4.3 Entrepreneurship and Income Distribution Dynamics

The main focus of this study is to explain the business cycle behavior of income distribution, in particular, the behavior of the top 5 percent income group, by incorporating entrepreneurial activities. We first examine the aggregate business cycle properties of the model economy in the presence of exogenous shifts in aggregate productivity,[39] and then we discuss the income distribution dynamics over the business cycles.

4.3.1 Key Business Cycle Statistics

The conventional set of business cycle statistics of the model economy along with the cyclical behavior of the U.S. aggregate data for the years 1968 to 2016 are reported in Table 7, showing the volatility of output and relative volatilities and cross correlations with output of the key aggregate variables. Although the volatility of output in the model is small compared to the actual output volatility, most statistics are similar to those found in the standard DSGE models as well as in the data: consumption is about 50 percent as volatile as output; investment is about three times as volatile as output; and so forth. Relative volatility across occupations is approximately well

[39]Even though our model is simulated at a quarterly frequency, we add up the values of the aggregate and individual variables to get simulated moments on an annual basis and then compare the simulated moments with the data. When the income distribution at an annual frequency is calculated from the simulated data, the individual data, such as income, are added up agent by agent over four quarters.

Table 8: Volatility of Income Share Relative to Output

	Quintiles				Percentile (%)	
	1st	2nd	3rd	4th	80-95	95-100
U.S. Data (CPS)	2.51	0.57	0.44	0.37	0.33	1.18
Benchmark Model	1.93	0.80	0.42	0.31	0.29	0.62

Note: The relative size of fluctuations of the income share is defined as the coefficients of variation of income shares to the standard deviation of output.

reproduced, in particular, showing the behaviors that the population share and the income share of entrepreneurs measured by the coefficients of variation are more volatile than those of workers.[40] Regarding income distribution, the model economy reproduces reasonably well the volatility and the countercyclicality of the Gini coefficient, even though the countercyclicality of the Gini coefficient of the data appears with a two quarter lag ($\rho(Gini_{t+2}, Y_t) = -0.54$) in the data but is observed contemporaneously in the model, showing a negative contemporaneous cross correlation with output. The population and the total income of entrepreneurs are more volatile than those of workers, respectively, and the population of workers is more procyclical than that of entrepreneurs, which are well reproduced in the benchmark model.

4.3.2 Cyclical Behavior of Income Distribution

Before discussing the cyclicality of the income distribution, as discussed in Castañeda, Díaz-Giménez, and Ríos-Rull (1998), it is also instructive to see the volatility of the income shares relative to output. Table 8 reports fluctuations in the income shares earned by the six income groups relative to aggregate output. Following Castañeda, Díaz-Giménez, and Ríos-Rull (1998), for income shares, we compute the coefficients of variation instead of the standard deviations. As already shown in Castañeda, Díaz-Giménez, and Ríos-Rull (1998), the relative volatility of the income shares shows the hook-shaped pattern over the income distribution in the U.S. data. The income share earned by the lowest quintile is the most volatile, and then the volatility of the income share decreases monotonically as we move towards the upper tail of the distribution: the relative volatility of income share for the first income group is 2.51, while that for the 80th to 95th percentile group is 0.33. However, it increases again for the top income group: the relative volatility of income share earned by the top earners is 1.18. The benchmark model economy broadly replicates the hook-shaped pattern of the relative volatility of the income shares over the income distribution. The model economy generates the highest value of the relative volatility of the income share earned

[40] The volatilities of population shares and income shares are measured by the coefficients of variation.

Table 9: Contemporaneous Correlation of Income Share with Output

	Quintiles				Percentile (%)	
	1st	2nd	3rd	4th	80-95	95-100
U.S. Data (CPS)						
All Households	0.52	0.46	0.41	-0.25	-0.48	-0.05
Excluding Entrepreneur	0.54	0.48	0.23	-0.35	-0.42	-0.32
Benchmark Model						
All agents	0.72	0.56	0.24	-0.19	-0.35	-0.07
Excluding Entrepreneur	0.71	0.57	0.17	-0.25	-0.23	-0.32
Model without Entrepreneurial Choice	0.85	0.79	0.63	-0.15	-0.62	-0.58

Note: "Excluding entrepreneurs" means the correlations of income shares with output after dropping entrepreneurs from the original sample, keeping the income groups fixed. "Model without entrepreneurial choice" refers to a heterogeneous agent model with only two occupational choices, workers and non-employed workers.

by the lowest quintile (1.93): it decreases monotonically until it reaches to 0.29 for the 80th to 95th percentile group; and it increases to 0.62 for the top five percent.

Our main interest is in the cyclical behavior of the income distribution. Table 9 reports the contemporaneous cross correlations of income shares with output for the six income groups in the benchmark model and compares these with the data. The benchmark economy successfully matches the cyclical behavior of the income shares with the U.S. data. The contemporaneous correlations of the income shares of the first, second, and third quintiles with output are 0.72, 0.56, and 0.24, respectively, in the model economy, and 0.52, 0.46, and 0.41, respectively, in the data. Therefore, the model economy successfully reproduces the procyclicality of the income shares of the three low-income quintiles. The intuitive explanation is as follows. When a positive aggregate shock occurs, more workers participate in the labor market (an increase in the extensive margin). Most of the new employment occurs in the low-income groups, which results in an increase in the income share of the low income quintiles.[41] On the contrary, rich households lose their shares of the total income since households in the high-income groups are already almost fully employed. Hence, the benchmark economy reasonably replicates the countercyclical behaviors of the fourth quintile and 80-95 percentile, making the contemporaneous correlations of the income shares with output -0.19 and -0.35, respectively, which are consistent with the U.S. data (-0.25 and -0.48, respectively).

One of the main contributions of our work is that the model economy successfully reproduces the acyclical behavior of the income share of the top five percent, which has not been successfully

[41] Most of the households in the lowest income quintile are non-employed, but the fraction of employment as workers increases in the higher income groups, as shown in Table A.4.

explained in previous studies, such as Castañeda, Díaz-Giménez, and Ríos-Rull (1998) and Heer (2013). The contemporaneous correlation of the income share of the top five percent is -0.05 in the data, while the corresponding correlation coefficients in the previous studies turned out to be strongly negative, reflecting a decrease in the income share of the top income group during expansions, as explained above. Our benchmark model economy with an entrepreneurial choice produces a correlation of -0.07, which is very close to the data.[42] We think that introducing the additional occupational choice of entrepreneurship is a key factor in generating the acyclical behavior of the income share of the top five percent.

Role of Entrepreneurship To examine the role of entrepreneurship in the cyclical property of the income shares earned by the top income earners, we calculate the correlations of income shares with output after dropping entrepreneurs, keeping the income groups fixed. The second row of the second panel in Table 9 reports the cross correlations of the income shares in the six income groups with output, after excluding entrepreneurs from the total sample in the benchmark model. Consistent with what we observe in the data, it is very suggestive that the correlation of the income share of the top five percent income group with output becomes strongly negative (-0.32) when excluding entrepreneurs. Therefore, we think that entrepreneurial activities play an important role in generating the acyclical behavior of the income share of the top five percent income group.

Another way to see the role of entrepreneurial activities is to consider a counterfactual economy without an entrepreneurial choice. This counterfactual economy allows for individual labor productivity shocks, but not for entrepreneurial productivity shocks, and, hence, there are only two occupations in the model without an entrepreneurial choice: employment and non-employment. In this sense, this is essentially a similar specification as in Chang and Kim (2007). The equilibrium of this economy can be defined similarly to that of the benchmark model with the two occupational choices. Except for β, $\overline{h}_W$, and $\overline{h}_N$, the same parameter values are used for the model without an entrepreneurial choice. Regarding β, $\overline{h}_W$, and $\overline{h}_N$, the same calibration strategies used in the benchmark model are also applied to this counterfactual economy: β is searched to be consistent with an interest rate of 1 percent in a steady state; $\overline{h}_W$ is chosen to be consistent with 70 percent employment; and the home-produced good for a non-employed worker, $\overline{h}_N$, is set for the total home production to target the 30 percent of GDP in a steady state. The last row of Table 9 reports the simulation results of the model economy where there is no entrepreneurial choice. The model without an entrepreneurial choice shows a different result from the benchmark model economy but

[42] The income share of the top one percent earners in our model economy also shows an acyclical behavior, which is consistent with the CPS.

Table 10: Decomposition of Cyclical Property of Income Share: Benchmark Model

	Quintiles				Percentile (%)	
	1st	2nd	3rd	4th	80-95	95-100
Correlation of Income Share with Output						
Worker	0.78	0.51	0.10	-0.07	-0.39	-0.49
Entrepreneurs	0.30	-0.06	0.18	-0.02	0.05	0.25
Non-employed Worker	0.19	-0.04	0.03	-0.12	-0.03	0.17
Correlations of Population Share with Output						
Worker	0.78	0.28	-0.11	-0.07	-0.11	-0.36
Entrepreneur	0.19	-0.14	0.17	0.01	0.14	0.26
Non-employed Worker	-0.79	-0.24	0.04	-0.10	-0.01	0.20
Correlation of Relative Average Income with Output						
Worker	0.59	0.61	0.46	-0.38	-0.60	-0.44
Entrepreneur	0.50	0.30	0.10	-0.19	-0.37	-0.03
Non-employed Worker	0.72	0.51	-0.10	-0.17	-0.15	-0.08

Note: Refer to the Notes in Table 2.

a similar result with the instance that entrepreneurs are excluded from the total sample in the benchmark model: the contemporaneous correlation of the income share of the top five percent is -0.58 in the economy with no entrepreneurial choice. From this exercise, we can conclude that entrepreneurial activities play a crucial role in generating the acyclical behavior of the income share for the top income earners.

Decomposition We also examine the decomposition of the cyclical behaviors of the income shares to take a close look at the role of entrepreneurship in the cyclical property of the income shares. Table 10, which is a counterpart of Table 2 in Section 2, summarizes the cross correlations of income shares, population shares, and relative average income with output in the two dimensions: the three occupations and the six income groups. According to the upper panel in Table 10, the correlations of the income shares with output are strongly positive for workers in the first three income groups, while they are negative in the 80th to 95th percentile group and in the top five percent. In other words, during economic expansions, the income shares for workers in the lower income quintiles increase while the income-rich lose their shares. This finding is consistent with the empirical findings discussed in Table 2. The middle and bottom panels of Table 10 show the relative importance between the extensive margin (population share) and the intensive margin (relative average income) in accounting for the cyclical behavior of income shares across the occupations and the income groups. As observed from the empirical findings in Table 2, both

factors are equally crucial for the income share dynamics for workers: most of the correlations for population shares and relative average income of workers are positive in the low-income quintiles but negative in the high-income groups.

Importantly, entrepreneurs seem to play a crucial role in explaining the acyclical behavior of the income share of the top five percent income earners by offsetting a reduction in the income share of the top income earners from the workers' side. Consistent with the empirical findings in the CPS, the income share of entrepreneurs in the top income group is positively correlated with output (the correlation is 0.25), which contrasts to that of workers (the correlation is -0.49). As seen in the middle panel of Table 10, there is a procyclical movement in the population of entrepreneurs in the top five percent but a countercyclical behavior of workers' population over the business cycle: the cross correlation with output of the population share of the top five percent is 0.26 for entrepreneurs, while it is -0.36 for workers.[43] This implies that, in an economic expansion, some workers at the top may switch to entrepreneurs.[44] Specifically, more households participate in economic activities as workers or entrepreneurs during expansions, but more people are involved in entrepreneurial activities, including switching from workers to entrepreneurs in the top income group, which offsets a decline in the income share of the top income earners from the workers' side. Hence, this transition activity between the two occupations at the top produces the acyclical behavior of the income share of the top income earners. This result is also consistent with the empirical findings shown in Table 2.

5 Discussions

5.1 Intuition from a Static Model

Why do workers want to be entrepreneurs during booms? To answer this question, we consider a simple model of the occupational choice. For analytical tractability and a clear illustration, we build a static economic environment. By using this simple model, we can characterize how a change in the aggregate productivity affects the selection into employment and entrepreneurship for households.

Suppose that a household has labor productivity, x, and entrepreneurial productivity, e, and

[43] Decomposing the behavior of the income share of entrepreneurs in the top five percent income group, the effect of the procyclical population share (the extensive margin) for entrepreneurs seems to dominate the behavior of the relative average income (the intensive margin) in that the relative average income shows the acyclical behavior with a correlation coefficient, -0.03. This finding is also consistent with what is observed in the data.

[44] Or, a relatively large portion of entrepreneurs from other lower income groups move up to the top, but we think that the transition between workers and entrepreneurs within the income group is more active and important.

holds assets, a. As in the benchmark model, there are a corporate sector and an entrepreneurial sector. It is assumed that the wage, w, is determined the corporate sector, and the production technology in the corporate sector is represented by the Cobb-Douglas production function, i.e., $F(K, L, z) = zK^{1-\alpha}L^{\alpha}$, where z aggregate productivity shock, and K and L are capital and effective labor used in the corporate production, respectively. For simplicity, we also assume that an aggregate productivity shock positively affects effective labor used in the corporate production, i.e.,

$$\frac{dL(z)}{dz} > 0. \tag{19}$$

The above condition implies that some non-employed households are exogenously employed during expansions. This simplicity assumption of exogenous transition between employment and non-employment over the business cycles allows us to focus on the key mechanism of an endogenous transition between employment and entrepreneurship in response to a change in the aggregate productivity.

If a household decides to work as an employed worker, it earns wx as wage earnings. Given the level of asset holdings, a worker can consume the sum of labor income and wealth,

$$c^w = wx + a, \tag{20}$$

where c^w denotes worker's consumption. In order to make the problem analytically tractable, it is assumed that entrepreneurs cannot borrow, the capital depreciation rate is zero, and the entrepreneurial production follows a constant-returns-to-scale (CRS) technology:

$$f(k, l, z, e) = z(ek)^{1-\alpha}l^{\alpha}, \tag{21}$$

where k and l are capital and outside labor, respectively. As in the benchmark model, we assume that an entrepreneur is required to hire outside labor. Since borrowing is not allowed for entrepreneurs, $k = a$. If a household with entrepreneurial ability, e, decides to be an entrepreneur, it earns π as profit income. Accordingly, an entrepreneur can consume the sum of profit income and asset holdings,

$$c^e = \pi + a, \tag{22}$$

where c^e denotes entrepreneur's consumption. Note that π can be written as:

$$\pi = \Delta w^{\frac{\alpha}{\alpha-1}} z^{\frac{1}{1-\alpha}} ea, \tag{23}$$

where $\Delta = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} > 0$. With the standard assumptions of the utility function for households (monotonicity and concavity), the occupational decision of a household will be made by comparing wage income, wx, and profit income, π.

Now let us illustrate how a change in the aggregate productivity, z, affects the transition between employment and entrepreneurship. Define the entrepreneur's income relative to wage income, $\Upsilon(z, \frac{e}{x}, a)$, as:

$$\Upsilon(z, \frac{e}{x}, a) = \frac{\pi(z, e, a)}{w(z)x} = \Delta \left(\frac{e}{x}\right) \left(\frac{z}{w(z)}\right)^{\frac{1}{1-\alpha}} a. \tag{24}$$

A household becomes an entrepreneur if $\Upsilon(z, \frac{e}{x}, a) > 1$. Two interesting findings emerge regarding the relative income, $\Upsilon(z, \frac{e}{x}, a)$. First, $\Upsilon(z, \frac{e}{x}, a)$ is an increasing function of the relative productivity, $\frac{e}{x}$, and the level of asset holdings, a. Second, in response to an increase in aggregate productivity, z, a household with $\frac{e}{x}$ and a given has a higher probability to be an entrepreneur as long as

$$\frac{\partial \Upsilon(z, \frac{e}{x}, a)}{\partial z} > 0. \tag{25}$$

The above condition holds if $\frac{\partial\left(\frac{z}{w(z)}\right)}{\partial z} > 0$. Since the wage rate is assumed to be determined in the corporate sector, $\frac{z}{w(z)}$ can be expressed as:

$$\frac{z}{w(z)} = \frac{1}{\alpha} \left(\frac{L(z)}{K}\right)^{1-\alpha}. \tag{26}$$

With the aggregate capital K fixed and under Condition (19), $\frac{\partial\left(\frac{z}{w(z)}\right)}{\partial z} > 0$ and, thus, $\frac{\partial \Upsilon(z, \frac{e}{x}, a)}{\partial z} > 0$, which implies that more households will be entrepreneurs with a higher z. Intuitively, an increase in aggregate productivity leads to an increase in employment in the corporate sector (Condition (19)), which does not increase wages $w(z)$ proportionally to the aggregate shock z. This implies that $\frac{z}{w(z)}$ increases, which, in turn, increases the income for entrepreneurs relative to workers. Therefore, from this analysis with the simple static model, we can conclude that during economic expansions, more workers become entrepreneurs, and this transition is more active for wealthy households with a relatively high entrepreneurial productivity.

5.2 Role of Collateral Constraints

We next discuss the role of collateral constraints in the income distribution dynamics. Table 11 shows the results for three cases: i) an entrepreneur cannot borrow for his entrepreneurial production, i.e., $\theta = 1.0$, ii) an entrepreneur can borrow up to 100 percent of his asset holdings, i.e.,

Table 11: Role of Collateral Constraint

	Quintiles				Percentile (%)		Total
	1st	2nd	3rd	4th	80-95	95-100	
$\theta = 1.0$							
Within-group Entrepreneur's Income Share	2.5	6.8	12.3	13.7	15.1	28.5	17.1
Within-group Entrepreneur's Pop. Share	1.6	6.5	12.1	13.6	14.8	24.0	10.2
Correlation of Income Share with Output	0.71	0.64	0.41	-0.07	-0.36	-0.27	
$\theta = 1.5$ *(Benchmark)*							
Within-group Entrepreneur's Income Share	2.7	7.2	12.5	13.8	16.6	39.1	20.8
Within-group Entrepreneur's Pop. Share	1.7	6.8	12.2	13.7	16.2	30.6	10.8
Correlation of Income Share with Output	0.72	0.56	0.24	-0.19	-0.35	-0.07	
$\theta = 2.0$							
Within-group Entrepreneur's Income Share	2.7	7.3	12.2	13.9	17.4	46.0	23.3
Within-group Entrepreneur's Pop. Share	1.7	6.8	11.9	13.7	17.0	35.2	11.1
Correlation of Income Share with Output	0.73	0.55	0.11	-0.23	-0.38	0.03	

$\theta = 2.0$, and iii) the benchmark case, i.e., $\theta = 1.5$. According to the upper panel of Table 11, when the credit market in the entrepreneurial production sector is completely shut down ($\theta = 1.0$), the steady state income and population shares for entrepreneurs decrease: the income share falls by around 4 percent points, and population share falls by 0.6 percent point. Not surprisingly, the top five percent is the income group most affected by the change in the collateral constraints, while other income groups remain relatively unchanged. The steady state within-group population share for entrepreneurs in the top income group decreases by around 7 percent points, and their within-group income share also falls by around 11 percent points. On the other hand, when credit constraints are loosened in the entrepreneurial production ($\theta = 2.0$), the income and population shares for entrepreneurs in a steady state increase, and these increases are mostly driven by entrepreneurs in the top five percent income group (see the bottom panel of Table 11).

As far as the cyclicality of the income distribution is concerned, the role of entrepreneurship for the acyclical behavior of the income share of the top income earners seems to be limited when the collateral constraints are tightened ($\theta = 1.0$) since entrepreneurs borrow less, so inflows from salary workers to entrepreneurs may decrease at the top. Accordingly, the decreased income share for entrepreneurs generates countercyclical behaviors of income share for the top income earners, making the correlation of their income share with output -0.27. However, when the collateral constraints are relaxed ($\theta = 2.0$), the correlation of the income share for top income earners with output is almost zero (0.03).

In summary, more severe financial frictions delay the picking up of entrepreneurial activity in booms because it will take longer for households to accumulate enough wealth to make entrepreneurship profitable. Hence, we can conclude that financial frictions play a crucial role in accounting for the cyclicality of the income distribution for the top income earners.

6 Concluding Remarks

This study attempts to explain the cyclical behavior of the income distribution by developing a heterogeneous agent general equilibrium model with entrepreneurial choices. We find the role of entrepreneurs significant in explaining the cyclical behavior of the income share in the income groups. The model economy successfully reproduces the acyclical behavior of the income share of the top five percent income earners as well as the overall cyclicality of income shares of the income groups. During expansions, relatively more households at the bottom of the income distribution newly begin employment as workers, which increases the income shares of households at the bottom of the income distribution and lowers the income shares of rich workers. At the top, however, relatively more people become entrepreneurs, which offsets the decline in the income share of the high-income earners from the workers' side.

Even though our model has some nice features such as adding entrepreneurial activities to the heterogeneous agent DSGE model, the model abstracts from risk aspects of entrepreneurial activities. In practice, entrepreneurial projects are inherently risky in the sense that an entrepreneur should be subject to the risk of losing his/her wealth invested in the business. Incorporating risky investment projects of entrepreneurs and risk behavior of agents would be possible directions for future extensions.

References

Abraham, A., and K. White (2006): "The Dynamics of Plant-Level Productivity in U.S. Manufacturing," Working Papers 06-20, Center for Economic Studies, U.S. Census Bureau.

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics. Vol. 109, No. 3, 659-684.*

Autor, D. H., and D. Dorn (2013): "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market," *American Economic Review*, 103(5), 1553–97.

Autor, D. H., L. F. Katz, and M. S. Kearney (2008): "Trends in U.S. Wage Inequality: Revising the Revisionists," *Review of Economics and Statistics*, 90(2), 300–323.

Buera, F. J., J. P. Kaboski, and Y. Shin (2011): "Finance and Development: A Tale of Two Sectors," *American Economic Review*, 101(5), 1964–2002.

Buera, F. J., and Y. Shin (2013): "Financial Frictions and the Persistence of History: A Quantitative Exploration," *Journal of Political Economy*, 121(2), 221–272.

Cagetti, M., and M. De Nardi (2006): "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*, 114(5), 835–870.

Castañeda, A., J. Díaz-Giménez, and J.-V. Ríos-Rull (1998): "Exploring the income distribution business cycle dynamics," *Journal of Monetary Economics*, 42(1), 93 – 130.

Castro, R., G. L. Clementi, and Y. Lee (2015): "Cross Sectoral Variation in the Volatility of Plant Level Idiosyncratic Shocks," *The Journal of Industrial Economics*, 63(1), 1–29.

Chang, Y., and S.-B. Kim (2007): "Heterogeneity and Aggregation: Implications for Labor-Market Fluctuations," *American Economic Review, 97(5), 1939-1956.*

Chang, Y., S.-B. Kim, and F. Schorfheide (2013): "Labor Market Heterogeneity, Aggregation, and the Policy-(In)variance of DSGE Model Parameters," *Journal of the European Economic Association*, 11, 193–220.

Cooper, R. W., and J. C. Haltiwanger (2006): "On the Nature of Capital Adjustment Costs," *The Review of Economic Studies*, 73(3), 611.

Evans, D. S., and B. Jovanovic (1989): "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints," *Journal of Political Economy*, 97(4), 808–827.

Floden, M., and J. Lindé (2001): "Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?," *Review of Economic Dynamics 4, 406-437.*

Foster, L., J. Haltiwanger, and C. Syverson (2008): "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?," *American Economic Review*, 98(1), 394–425.

French, E. (2005): "The Effects of Health, Wealth, and Wages on Labor Supply and Retirement Behavior," *Review of Economic Studies, 72(2), 395-427.*

Giannelli, G. C., L. Mangiavacchi, and L. Piccoli (2012): "GDP and the value of family caretaking: how much does Europe care?," *Applied Economics*, 44(16), 2111–2131.

Guvenen, F., S. Ozkan, and J. Song (2014): "The Nature of Countercyclical Income Risk," *Journal of Political Economy*, 122(3), 621–660.

Heathcote, J., F. Perri, and G. L. Violante (2010): "Unequal we stand: An empirical analysis of economic inequality in the United States, 1967-2006," *Review of Economic Dynamics*, 13(1), 15 – 51, Special issue: Cross-Sectional Facts for Macroeconomists.

Heer, B. (2013): "A note on the cyclical behaviour of the income distribution," *OECD Journal: Journal of Business Cycle Measurement and Analysis*, 2013(1), 1–7.

Huggett, M. (1993): "The risk-free rate in heterogeneous-agent incomplete-insurance economies," *Journal of Economic Dynamics and Control, Elsevier, vol. 17(5-6), 953-969.*

Hurst, E., and B. W. Pugsley (2011): "What Do Small Businesses Do?," *Brookings Papers on Economic Activity*, 41(2 (Fall)), 1–70.

Katz, L. F., and K. M. Murphy (1992): "Changes in Relative Wages, 1963-1987: Supply and Demand Factors," *The Quarterly Journal of Economics*, 107(1), 35.

Krusell, P., and A. A. Smith (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy, 106(5): 867-96.*

Kydland, F. E., and E. C. Prescott (1982): "Time to build and aggregate Fluctuations," *Econometrica, 50, 1345-79.*

Lee, Y., and T. Mukoyama (2015): "Productivity and employment dynamics of US manufacturing plants," *Economics Letters*, 136, 190 – 193.

Lusardi, A., and O. S. Mitchell (2014): "The Economic Importance of Financial Literacy: Theory and Evidence," *Journal of Economic Literature*, 52(1), 5–44.

Midrigan, V., and D. Y. Xu (2014): "Finance and Misallocation: Evidence from Plant-Level Data," *American Economic Review*, 104(2), 422–58.

Parker, J. A., and A. Vissing-Jorgensen (2010): "The Increase in Income Cyclicality of High-Income Households and Its Relation to the Rise in Top Income Shares," *Brookings Papers on Economic Activity*, 43(2 (Fall)), 73–142.

Piketty, T. (2014): *Capital in the Twenty-First Century*. Belknap Press.

Quadrini, V. (2000): "Entrepreneurship, Saving, and Social Mobility," *Review of Economic Dynamics*, 3(1), 1 – 40.

Ríos-Rull, J.-V. (1997): "Computation of equilibria in heterogeneous agent models," Staff Report 231, Federal Reserve Bank of Minneapolis.

Takahashi, S. (2015): "Time-Varying Wage Risk, Incomplete Markets, and Business Cycles," *Discussion Paper No. 912, Institute of Economic Research, Kyoto University*.

Tauchen, G. (1986): "Finite state markov-chain approximations to univariate and vector autoregressions," *Economics Letters*, 20(2), 177 – 181.

Terajima, Y. (2006): "Education and Self-Employment: Changes in Earnings and Wealth Inequality," Staff Working Papers 06-40, Bank of Canada.

Vargha, L., R. I. Gal, and M. O. Crosby-Nagy (2017): "Household production and consumption over the life cycle: National Time Transfer Accounts in 14 European countries," *Demographic Research*, 36(32), 905–944.

Yurdagul, E. (2017): "Production complementarities and flexibility in a model of entrepreneurship," *Journal of Monetary Economics*, 86(Supplement C), 36 – 51.

Appendix

A Data

A.1 CPS

For information on income and occupation status, we use the ASEC of the CPS which covers the years from 1968 to 2016. The ASEC of the CPS contains detailed questions covering economic characteristics surveyed in every March and provides estimates based on a survey of around 65,000 households. The basic unit of observations for the CPS is a household. Data for the CPS are downloaded from Integrated Public Use Microdata Series (IPUMS).[45]

A.1.1 Sample Selection

We describe the details of how we clean the raw data of the CPS. We start by dropping persons that have "Not In Universe" values for variables regarding income and employment status. We exclude the households that are armed forces. We then drop the households that do not have a "head." Next we aggregate various kinds of income for all a household members in a household to get a household level data. After the sample selection process, the average sample size for the households is around 60,000.

A.1.2 Definition of Income

Information on income in the CPS sample refers to income received during the previous calendar year. We define a household income as the sum of labor income, self-employment income, and net asset income before tax for all persons of a household. We exclude private transfers and public transfers to be consistent with the definition of income in the model economy. Details on the definitions of each income categories are as follows.

- Labor income: wage and salary income (Variable name: INCWAGE) for 1968-1987, and the sum of earnings from longest job (INCLONGJ) and earnings from other work included wage and salary earnings (OINCWAGE) for 1988-onward.
- Net asset income: income from interest, dividends, and rent (INCIDR) for 1968-1975, income from dividends, rent, and trusts (INCDRT) for 1976-1987, and the sum of income from interest (INCINT), dividends (INCDIVID), and rent (INCRENT) for 1988-onward.

[45] https://cps.ipums.org/cps/index.shtml.

- Self-employment income: the sum of farm income (INCFARM) and nonfarm business income (INCBU.S.) for the entire sample period.

A.1.3 Top-coding

Top-coding is an important issue in this paper since the behavior of the top income households is a main interest. Thresholds of top codes of the CPS vary across income categories and across sample periods. For the years prior to 1996, we deal with top-coded samples by using the adjustment factor of 1.5: we multiply top-coded income values by 1.5, which is widely used in the income distribution literature. For example, Katz and Murphy (1992) and Autor, Katz, and Kearney (2008) use an adjustment factor of 1.5 to deal with the top codes in the CPS and Autor and Dorn (2013) also employ an adjustment factor of 1.5 for the top-coded values in the American Community Survey (ACS) data. The Census Bureau started to address the top-coding issue since 1996. Starting in 1996, the top-coded individuals were divided into twelve groups depending on characteristics such as gender, race, and full time status and income values of top-coded individuals were replaced with the mean income within each group. In 2011, the Census Bureau shifted from the average replacement value system to a rank proximity swapping procedure.[46] We use the Census Bureau data unadjusted for the years starting from 1996 since we are interested in the behavior of the top five percent as a group rather than as an individual and our work is not subject to the treatment of the Census Bureau regarding top-coding issues.

A.1.4 Definition of Occupations

A households are classified into three categories according to occupational status: an entrepreneur, a worker, and a non-employed worker. The head of a household is defined as the variable of relationship to a household head (RELATE). To classify workers and non-employed works, we use a question indicating whether the respondent worked during the preceding week (EMPSTAT). We use self-employment income, defined above, to define the entrepreneurs. Definitions of the occupations in the CPS are as follows.

- Entrepreneur: a household in which at least one person in a household earn self-employment income.

- Worker: a household in which the head of a household worked during the preceding week, and no one in a household earns self-employment income.

[46]For more information regarding this procedure including top codes, replacement values, and swap values for income components, refer to the website, https://cps.ipums.org/cps/topcodes_tables.shtml.

- Non-employed worker: a household in which the head of a household did not work during the preceding week, and no one in a household does not earn self-employment income.

A.2 PSID

We use the PSID samples for period 1983-1989 to obtain the income and wealth distribution because the years are around the median of the sample periods of CPS. We use the PSID surveys to calculate the income share, the population share, and the relative average income across the income groups and the average one-year transition probabilities for income across occupations for Table A.1. The number of families in the PSID is around 6,000 per year. We summarize the PSID variables that we use for income, wealth, and the definitions of the three occupations.

- Income: total taxable income of head and wife
- Wealth: the sum of values of six asset types, net of debt value, and value of home equity
- Occupation: 1) Did you (or anyone else in the family there) own a business or have a financial interest in any business enterprise?, and 2) work for someone else, yourself, or what.

A.3 Aggregate Data

This subsection explains the data sources used for the moments provided in Table 7. All data, except for the employment rate for workers and entrepreneurs, are drawn from the St. Louis Fred economic database for the sample years 1968 to 2016.

- Output (Y): real gross domestic product minus real government consumption expenditures and gross investment (Fred ID: GDPCA-GCECA).
- Consumption (C): sum of real consumption of nondurable goods and services which are deflated by the implicit price deflators for nondurable goods and services, respectively (Fred ID: PCND/DNDGRD3A086NBEA + PCESV/DSERRD3A086NBEA).
- Investment (I): sum of real consumption of durable goods and real gross private domestic investment deflated by the implicit price deflators for durable goods and gross private domestic investment, respectively (Fred ID: PCDGA/DDURRD3A086NBEA + GPDIC1/A006RD3A086NBEA).
- Population share for workers (PW): CPS 1968-2016.
- Population share for entrepreneurs (PE): CPS 1968-2016.
- Gini: income Gini ratio of families (Fred ID: GINIALLRF).

B Computational Procedures

In this subsection, we summarize the computational methods and procedures for the benchmark economy.

B.1 Steady-state Economy

We use the algorithm suggested by Ríos-Rull (1997) to find the stationary measure, $\mu_s(a, x, e)$. The steps are as follows.

1. *Setting guess for endogenous parameters:* We start with the initial guess for endogenous parameters.

2. *Constructing grids for a, x, and e:* The numbers of a, x, and e grids are denoted by n_a, n_x, and n_e, respectively. We choose $n_a = 101$, $n_x = 13$, and $n_e = 13$. The range of the asset holding is $[-2, 300]$, and asset grids are not equally spaced. $\hat{s}\,(\equiv \ln s)$, is equally spaced in the range of $[-3\sigma_{\hat{s}}, 3\sigma_{\hat{s}}]$, where $\sigma_{\hat{s}} = \sigma_s/\sqrt{1-\rho_s^2}$ and $s \in \{x, e\}$.

3. *Approximating the transition probability matrices for x and e* : We use the method developed by Tauchen (1986) to approximate the transition probability matrices for x and e, $Q_x(x'|x)$ and $Q_e(e'|e)$, respectively.

4. *Solving the individual value functions:* Given the parameters, we solve a set of value functions, $\{V_W, V_E, V_N, V\}$, on each grid point of the individual states. In this step, we obtain the optimal decision rules for asset holding, $a'(a, x, e)$, consumption, $c(a, x, e)$, effective labor used for entrepreneurial production, $l(a, x, e)$, and investment on entrepreneurial production, $k(a, x, e)$.

5. *Obtaining time-invariant measures:* We obtain the time-invariant measures, $\mu_s(a, x, e)$. Using cubic spline interpolation, obtain the new value functions with fine grid points for assets. Compute the new optimal decision rules by using the new value functions. Compute the time-invariant measures by using the new optimal decision rules and the transition probability matrices for x and e.

6. *Updating the parameters:* Compute the aggregate variables by using $\mu_s(a, x, e)$. If the obtained values are close enough to the targeted values, the steady-state economy is found. Otherwise, we update the parameters and go back to Step 4.

B.2 Dynamic Economy

For an equilibrium with aggregate fluctuations, we follow Krusell and Smith (1998), who suggest that a very high precision can be obtained by approximating the type distribution of individuals using a mean asset, the first moments of the distribution. The steps are as follows.

1. *Constructing K and z grids:* The grids for individual state variables are the same with those used in the steady state economy. The number of grids for K and z are denoted by n_K and n_z, receptively. We chose $n_K = 7$ and $n_z = 7$. The range of K is $[0.8K_s, 1.2K_s]$, where K_s is the steady-state mean capital. The logged aggregate productivity, $\hat{z}\,(\equiv \ln z)$, is equally spaced in the range of $[-3\sigma_{\hat{z}}, 3\sigma_{\hat{z}}]$, where $\sigma_{\hat{z}} = \sigma_z/\sqrt{1-\rho_z^2}$.

2. *Parameterizing the forecasting function*: We use the following log-linear functional form for forecasting functions of K' and w:

$$\ln K' = b_0 + b_1 \ln K + b_2 \ln z,$$

$$\ln w = d_0 + d_1 \ln K + d_2 \ln z.$$

   We set the initial guess for the coefficients in the forecasting functions.

3. *Solving the optimization problem for the individual households*: Using the forecasting functions obtained in the previous step, we solve the individual optimization problem to obtain a set of value functions and optimal decision rules.[47]

4. *Implementing the simulation:* Using the forecasting functions and value functions, solve the optimization problem individual for 3,500 periods with fine grid points for assets given the initial values for K, z and $\mu(a, x, e)$. The detailed steps are as follows.

   (a) Set initial values for K, z and $\mu(a, x, e)$.

   (b) Given aggregate variables, obtain the value function $\widehat{V}(a, x, e)$ for fine asset grids by using the value function obtained in Step 3 and the forecasting the function for K'.

   (c) Set $\widehat{w}$, as a guess for the wage rate, and under $\widehat{w}$, obtain the decision rule for investment on entrepreneurial production, $k(a, x, e)$, and effective labor used for the entrepreneurial firm, $l(a, x, e)$, by using the value function $\hat{V}(a, x, e)$.

[47] Given the wage rate w, the real interest rate, r, is computed from the corporate firm's profit maximization.

(d) Check labor market clearing: $L = z\{(1-\alpha)/\hat{w}\}^{1/\alpha}\hat{K} = \int x d\mu_W - \int l(a,x,e)d\mu$, where $\hat{K} = K - \int k(a,x,e)d\mu$. If the labor market clears, then go to the next step. Otherwise, reset $\hat{w}$, and go back to Step (c). The wage rate that the labor market clears is denoted by w^*.

(e) Using the value function $\hat{V}(a,x,e)$ and w^*, obtain the decision rules.

(f) Compute aggregate variables by using $\mu(a,x,e)$ and the decision rules.

(g) Obtain the next-period distribution $\mu'(a,x,e)$ by using transition probabilities for x and e and the decision rule for assets.

5. *Obtaining new values for coefficients*: We obtain the new set of coefficients in the forecasting functions from an OLS regression with the 3,000 simulated data for K', z, and w^*.[48] If the new coefficients obtained are close enough to the previous coefficients, we are done. Otherwise, we update the set of coefficient and go back to Step 3. We check the goodness of fit for the forecasting functions with R^2. The forecasting functions provide high accuracy:

$$\ln K' = 0.12263 + 0.93869 \ln K + 0.10930 \ln z,\ R^2 = 0.9997,$$

$$\ln w = 0.16756 + 0.34867 \ln K + 0.92405 \ln z,\ R^2 = 1.0000.$$

C Other Findings

C.1 Entrepreneurship and Income Mobility

Another interesting finding of this study is the role of entrepreneurship in income mobility. Quadrini (2000) shows that entrepreneurs experience a higher upward mobility in wealth distribution than workers. Since our primary interest is on the behavior of the income distribution, it is interesting to examine how the heterogeneous DSGE model with entrepreneurial activities performs in explaining income mobility across income groups. Table A.1 reports the one-year income transition probability matrices among each of the occupational statuses averaged over the period 1983-1989 from the PSID data.

Table A.1 reports one-year income transition probability matrices of four groups of households across the six income groups, averaged over the period 1983-1989 with the PSID data. The first group consists of *staying entrepreneurs*, households that are entrepreneurs in the current year as

[48] We drop the first 500 periods of 3,500 periods to eliminate the influence of the arbitrary choice of initial values.

well as in the previous year. *Staying non-entrepreneurs* are defined as the families that are workers or non-employed in the current year as well as in the previous year. *Switching entrepreneurs* are the families that were entrepreneurs in the previous years but switch to workers or non-employed workers in the current year. Similarly *switching non-entrepreneurs* are defined as the families that were workers or non-employed in the previous years but switch to entrepreneurs in the current year. In the first five income groups (from the first quintile to the 80-95 percentile), the percentage of moving to higher income groups is greater for staying entrepreneurs than for the households switching from entrepreneurs to non-entrepreneurs in the current year. In the first quintile, for example, about 59.1 percent (the sum of 34.4, 11.5, 8.3, 1.7, and 3.2 percent) of the households that were entrepreneurs in the previous year move to higher income groups when they stay as entrepreneurs in the current year, while only 30.7 percent (the sum of 27.5, 1.3, 1.9, 0.0, and 0.0 percent) of the households move to higher income groups when they switch from entrepreneurs to non-entrepreneurs. This feature is also observed for all the other income groups. The probability of falling to lower income groups is also much less for staying entrepreneurs than for switching entrepreneurs in the second quintile through the top five percent group. In addition, the probability of staying in the top five percent for staying entrepreneurs is much higher than for switching entrepreneurs. Therefore, entrepreneurial activities are more likely to provide an opportunity to earn a higher income.

The transition probabilities across the income groups for the households that were not entrepreneurs in the previous year are reported in the bottom panel of Table A.1. It is also evident that the households switching from non-entrepreneurs to entrepreneurs are more likely to move to higher income groups, when compared to staying non-entrepreneurs in the five income groups from the first quintile to the 80-95 percentile. A households switching from non-entrepreneur to entrepreneur in the top five percent income group in the previous year have a slightly higher probability (74.5 percent) of staying in the same income group than staying non-entrepreneurs (73.3 percent). The probability of falling to lower income groups is less for switching non-entrepreneurs than for staying non-entrepreneurs for most of the income groups. In summary, the statistics from the PSID 1983-1989 show that there is a noticeable difference in the income mobility of entrepreneurs and non-entrepreneurs across the income groups: entrepreneurship provides a higher probability of upward mobility to higher income groups than non-entrepreneurship. This finding is in line with Quadrini (2000), who focuses on the role of entrepreneurs in wealth mobility.

Table A.2 reports the one-year transition probabilities of staying entrepreneurs, switching entrepreneurs, staying non-entrepreneurs, and switching non-entrepreneurs across the income groups, for the model economy. The transition probability matrices of the model are very similar to those

Table A.1: One-year Transition Probabilities for Income: PSID 1983-1989

	1st	2nd	3rd	4th	80-95	95-100	1st	2nd	3rd	4th	80-95	95-100
	Staying Entrepreneur						Switching Entrepreneur					
1st	41.0	34.4	11.5	8.3	1.7	3.2	69.3	27.5	1.3	1.9	0.0	0.0
2nd	11.1	55.9	22.0	6.8	3.2	1.0	23.1	54.1	19.6	2.3	0.9	0.0
3rd	3.4	14.7	45.8	25.5	8.2	2.4	5.2	24.4	49.6	15.9	4.9	0.0
4th	2.2	4.4	20.2	47.3	22.6	3.3	5.1	11.6	16.4	51.3	14.3	0.0
80-95	1.5	1.2	5.7	20.3	54.2	17.1	0.6	2.2	4.0	24.9	58.5	6.7
95-100	0.3	0.6	1.9	3.8	19.4	74.0	0.0	3.2	4.4	9.0	22.0	61.3
	Staying Non-entrepreneur						Switching Non-entrepreneur					
1st	85.2	13.0	1.3	0.3	0.1	0.0	53.3	38.8	7.5	0.4	0.0	0.0
2nd	13.8	65.5	17.9	2.4	0.3	0.1	12.9	59.7	21.3	4.1	1.9	0.0
3rd	2.2	15.8	63.5	16.9	1.4	0.1	3.4	20.3	52.2	21.1	3.1	0.0
4th	0.6	2.7	14.7	67.9	13.5	0.6	0.6	8.8	20.8	49.1	19.2	1.6
80-95	0.2	0.8	2.0	18.0	73.1	6.0	0.0	2.1	6.1	21.1	54.5	16.2
95-100	0.2	0.7	0.8	2.7	22.3	73.3	1.5	0.0	0.0	3.8	20.3	74.5

Note: The upper panel reports the transition probabilities across the income groups for the households that were entrepreneurs in the previous year, and the bottom panel reports the transition probabilities across the income groups for the households that were not entrepreneurs in the previous year.

of the data in Table A.1.

The model economy reproduces the feature that the probability of moving to higher income quintiles for staying entrepreneurs in each income group is higher than that of the households switching from entrepreneurs to non-entrepreneurs. For example, according to the upper panel of Table A.2, a household staying as an entrepreneur of the first quintile in the current period have a probability of 87.1 percent of moving to a higher income group, while the probability is 44.5 percent for the households that switch from entrepreneur to non-entrepreneur. The model economy also shows the behavior that the likelihood of falling to a lower income group is less for staying entrepreneurs than for the households switching to non-entrepreneurs in each of the income groups. As for the top five percent, the model economy replicates the fact that staying entrepreneurs are more likely to stay in the top five percent than are switching entrepreneurs. Among staying entrepreneurs, 79.2 percent of the households were successful in remaining in the top five percent, while only 24.4 percent of the households switching to non-entrepreneurs, in the top five percent, managed to stay in the same income group. The bottom panel of Table A.2 reports the transition probabilities for non-entrepreneurs in the previous year. The higher probability to move upward to higher income groups for the switching households from non-entrepreneurs to entrepreneurs than for staying non-entrepreneurs is successfully replicated in the model economy. Therefore, the model

Table A.2: One-year Transition Probabilities for Income: Benchmark Model

	1st	2nd	3rd	4th	80-95	95-100	1st	2nd	3rd	4th	80-95	95-100
	Staying Entrepreneur						Switching Entrepreneur					
1st	13.0	48.7	31.4	6.6	0.4	0.0	55.5	22.6	15.6	5.8	0.5	0.0
2nd	4.5	31.5	40.9	19.2	3.9	0.1	36.0	29.3	21.7	10.9	2.1	0.0
3rd	1.3	14.2	40.5	32.5	10.8	0.6	21.6	28.1	25.6	19.6	5.0	0.1
4th	0.2	3.7	21.9	40.8	29.4	4.0	9.4	21.9	23.9	28.2	16.0	0.7
80-95	0.0	0.6	5.0	20.6	48.3	25.5	1.4	13.1	19.0	27.5	33.6	5.5
95-100	0.0	0.0	0.2	1.6	19.0	79.2	0.0	3.0	9.6	22.3	40.7	24.4
	Staying Non-entrepreneur						Switching Non-entrepreneur					
1st	60.1	23.6	11.1	4.5	0.7	0.0	25.0	43.7	24.6	6.3	0.5	0.0
2nd	20.6	43.7	24.3	9.5	1.8	0.1	8.2	33.0	35.1	16.9	6.3	0.4
3rd	12.7	18.1	35.8	27.1	6.1	0.2	5.2	23.4	38.6	24.1	7.8	1.0
4th	8.0	9.6	18.8	39.1	23.2	1.2	2.0	11.5	31.7	39.1	14.7	1.1
80-95	3.1	5.8	8.2	22.5	48.5	11.9	0.2	3.7	12.4	36.3	41.6	5.8
95-100	0.1	2.6	4.0	9.2	31.8	52.3	0.0	0.4	1.4	8.0	46.1	44.1

Note: The upper panel reports the transition probabilities across the income groups for the households that were entrepreneurs in the previous year, and the bottom panel reports the transition probabilities across the income groups for the households that were not entrepreneurs in the previous year.

economy shows that involvement in entrepreneurial activities is related to upward movements over the income groups, which is, we think, particularly important for the cyclical behavior of the top five percent, since the fraction of entrepreneurs in this group is the largest among all the income groups.

C.2 Alternative Model with Heterogeneous Capital Returns

In our model, the households become entrepreneurs only when the entrepreneurial opportunity is better than the employment opportunity. Therefore, it is reasonable to think that the rate of return to entrepreneurs' capital is always higher than or equal to the market rate of return to capital. We get the similar results of income distribution dynamics described above in the heterogeneous DSGE model with entrepreneurial choices when we simply introduce exogenously heterogeneous capital returns across agents without entrepreneurial choices. We propose a heterogeneous rate of return to capital, simply proportional to the level of individual labor productivity. This feature may increase the inequality of wealth distribution and in turn, the role of capital income, which we think plays a role in explaining the cyclical behavior of income distribution over the business cycle, in particular, offsetting the cyclicality of labor income across the income quintiles. Even though the heterogeneous rate of return on capital across individuals is assumed to be a linear function

of the labor productivity level of each agent, it is not very counterfactual. Figure A.1 plots the relationship between the rate of return to capital and wage rate from the PSID 1994 over ten income deciles.[49] The rate of return and the wage rate are calculated as averages in each income decile.[50] Surprisingly, they are linearly related with a cross correlation coefficient, 0.88.[51]

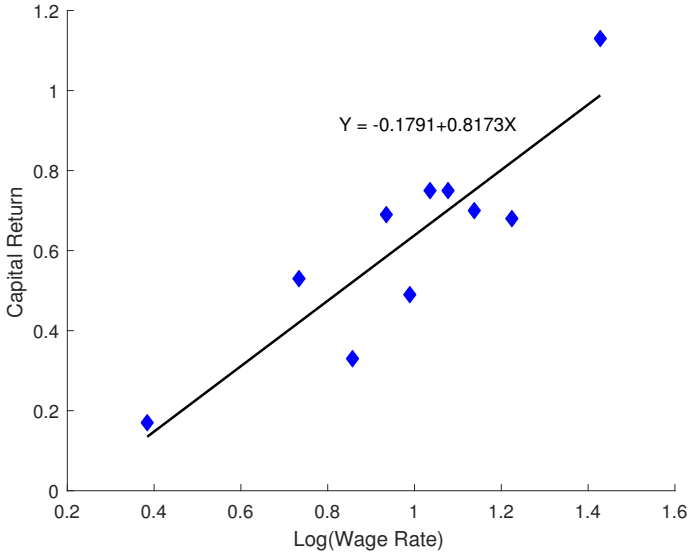

Figure A.1: Wage Rates and Capital Returns

Note: The wage rate is defined as the annual wage income divided by the annual working hours of the head of a household, and the capital return is defined as the ratio of the annual capital income to total wealth for each decile.

All other specifications are the same as in the models described above, except for two features: no entrepreneurial choice and heterogeneity in the rate of return to capital. The rate of return on assets each agent faces is assumed to depend on the agent's labor productivity level, x: $\widetilde{r} = \kappa x r$, where $\widetilde{r}$ is the rate of return on assets each individual faces and r is the rate of return on the aggregate capital. Regarding calibration, we determine the value of κ to satisfy the following equilibrium condition in the aggregate economy:

$$rK = \int \widetilde{r} a d\mu. \tag{A.1}$$

The model economy successfully reproduces the acyclical behavior of the income distribution

[49]The wage rate is defined as the annual wage income divided by the annual working hours of the head of a household, and the capital return is defined as the annual capital income over total wealth for each decile. We use the PSID 1994 survey because the year contains more information on capital incomes and family wealth than the PSID 1989.

[50]We take the average of the deciles after deleting the observations with zero assets or unemployment (typically with annual working hours under 100).

[51]The theoretical background for the idea of heterogeneity in the rate of return to capital across individuals can be found in Lusardi and Mitchell (2014) who survey the models with heterogeneity arising from human capital on financial knowledge.

for the top five percent income earners, with the correlation coefficient of their income share with output of -0.1. Since some parts of the entrepreneurs' income is treated as capital income and entrepreneurial opportunities are the most active in the top income group, the heterogeneous capital returns can be interpreted as a reflection of entrepreneurial activities.

D Additional Tables

Table A.3: Within-group Income Share of Each Occupation across Income Groups

	Quintiles				Percentile (%)		Total
	1st	2nd	3rd	4th	80-95	95-100	
Worker							
PSID	31.5	67.9	80.1	81.9	77.5	49.6	72.4
CPS	20.4	56.0	70.2	75.6	76.4	66.1	71.4
Benchmark Model	42.7	59.9	67.2	72.3	75.1	57.6	67.5
Entrepreneur							
PSID	3.5	6.0	8.9	12.6	19.6	46.7	21.0
CPS	8.5	12.7	13.7	14.8	16.6	26.7	17.5
Benchmark Model	2.7	7.2	12.5	13.8	16.6	39.1	20.8
Non-employed Worker							
PSID	65.0	26.1	11.0	5.5	2.9	3.7	6.7
CPS	71.1	31.3	16.1	9.7	7.0	7.2	11.1
Benchmark Model	54.7	32.9	20.4	13.9	8.3	3.4	11.8

Note: "With-in group Income Share" means the fraction of each occupation's income out of the total income in a specific income group. Therefore, the sum of the income shares of workers, entrepreneurs, and non-employed workers in each income group is 100.

Table A.4: Within-group Population Share of Each Occupation across Income Groups

	Quintiles				Percentile (%)		Total
	1st	2nd	3rd	4th	80-95	95-100	
Worker							
PSID	12.3	64.1	79.8	82.0	78.0	56.1	62.1
CPS	8.2	52.2	69.8	75.4	76.5	68.1	56.0
Benchmark Model	30.1	59.4	66.7	72.1	75.1	65.6	60.2
Entrepreneur							
PSID	1.4	5.7	8.8	12.4	19.0	40.7	10.5
CPS	3.5	12.6	13.7	14.7	16.5	25.0	12.6
Benchmark Model	1.7	6.8	12.2	13.7	16.2	30.6	10.8
Non-employed Worker							
PSID	86.3	30.2	11.4	5.7	3.0	3.2	27.4
CPS	88.2	35.3	16.5	9.8	7.1	6.9	31.4
Benchmark Model	68.2	33.8	21.1	14.2	8.6	3.8	28.9

Note: "With-in group Population Share" means the fraction of population for each occupation within an income group. Thus, the sum of the shares of the three occupations in each income group is 100.

Table A.5: Relative Average Income of Each Occupation across Income Groups

	Quintiles				Percentile (%)		Total
	1st	2nd	3rd	4th	80-95	95-100	
Entrepreneur/Worker							
PSID	0.98	0.99	1.01	1.02	1.04	1.30	1.71
CPS	1.06	0.94	1.00	1.00	1.01	1.10	1.09
Benchmark Model	1.10	1.05	1.02	1.01	1.02	1.44	1.71
Entrepreneur/Non-employed Worker							
PSID	3.41	1.21	1.05	1.05	1.06	1.08	8.33
CPS	3.43	1.14	1.03	1.02	1.02	1.04	4.01
Benchmark Model	1.95	1.09	1.06	1.03	1.06	1.41	4.68

Note: The relative average income means the ratio of the average income of the one occupation to the average income of the other.