THERMAL STRESSES IN ANISOTROPIC PLATES

A Thesis

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Master of Science

in

Nuclear Engineering

by
Prabh Nandan Singh
B.Sc., Panjab Engineering College, Chandigarh, India, 1967
August, 1972

ACKNOWLEDGEMENT

The author would like to express his appreciation to Dr. M. Sabbaghian for directing this research and serving as major advisor, Dr. John C. Courtney and other members of the examining committee for their assistance and guidance in the preparation of this thesis.

He also wishes to express his thanks to Mr. Ernest Hamilton, Jr., and the Louisiana State University Computer Research Center for their help in the preparation of his computer program.

TABLE OF CONTENTS

LIST OF FIGURES

NOTATION

T, T_0, T_1	Temperature distributions in the plate
a, b, $2h$	Length, width, and thickness of the plate, respectively
u, v	Displacements of a point (X, Y, Z) of the plate in X and Y directions, respectively
w	Deflection of middle surface of the plate
τ, τ^0, τ^1	Stress tensors corresponding to T, T_0, T_1 distribution
σ_x, σ_y, σ_z	Average normal stress in direction of the axis indicated by subscript
N_x, N_y	Normal forces per unit of length in X and Y directions, respectively
N_{xy}, Q_x, Q_y	Shear forces per unit of length in the planes XY, XZ, and YZ, respectively
a_{ij}, c_{ij}	Elastic constants (compliances and stiffnesses, respectively)
b_{ij}	Modified stiffnesses
α_i	Linear coefficients of thermal expansion
α_i^1	Modified linear coefficients of thermal expansion
ε_x, ε_y, γ_{xy}	Unit elongations in the X and Y directions, and shearing strain component in the XY plane, respectively
F	Stress function
Δh, Δk	Finite increments in the X and Y directions, respectively, for solving finite difference equations
Nx, Ny	Number of grid points in the X and Y directions, respectively
M_x, M_y, M_{xy}	Moments per unit of length of the stresses σ_x, σ_y, τ_{xy} about the middle surface of the plate, respectively.

ABSTRACT

Many materials used in the nuclear industry have anisotropic properties. This thesis investigates stresses within a simple body due to temperature variations. A computer program is presented which can be used to calculate thermal stresses and deflections in an anisotropic plate with all four edges free and having one plane of elastic symmetry. General fourth order polynomials are assumed to predict the correct temperature distributions. Stresses are first calculated for a two-dimensional (X and Y) temperature distribution with no deflection (in Z direction) allowed. Then a three-dimensional case with a linear temperature gradient in Z direction is considered. Superimposition of these solutions leads to a general treatment of the normal and shear stresses in the plate. The correct elastic constants for the material being considered are required as input to the program.

CHAPTER I

INTRODUCTION

Anisotropic materials play an important part in modern technology. These find applications in a number of areas such as the aerospace, missile, aircraft and nuclear industries. It is possible to produce an artificial anisotropy in isotropic materials. For example, corrugated plates and membranes can be made from elastically isotropic materials. Certain structures which are strengthened by ribbing also become anisotropic.

An elastic medium is isotropic if in all directions its elastic properties at all points are the same. It will be anisotropic if its elastic properties at any point in the medium are different in different directions. An anisotropic body may have one, two or three orthogonal planes of elastic symmetry passing through any point in the body. The plane of elastic symmetry is the plane of the body about which its elastic properties are symmetric. The body which has three orthogonal planes of elastic symmetry at each point is called orthogonally anisotropic, or simply, orthotropic. If there are sets of equivalent directions in a body which can be superimposed by a rotation of $2\pi/N$ about a certain axis, say Z-axis, then this is an axis of elastic symmetry of order N. A plane normal to an axis of elastic symmetry of an infinitely large order ($N \to \infty$) is a plane of isotropy. A body having this property is called a transversely anisotropic body.

This is also described as a transversely isotropic body.

Under certain load conditions, such as thermal or external forces, an elastic body undergoes deformations or shape changes. Stresses, that is, loads per unit area, are related to these deformations by the well known Hooke's law for elastic materials. In the case where there is no motion of the body upon application of the forces, the body as a whole must be in equilibrium. If a small volume element in the body is considered, certain relations known as equilibrium equations arise. Also, any body has certain constraints acting on its boundaries. Using Hooke's law, the equilibrium equations and the boundary conditions simultaneously could lead to a solution for the distribution of the stresses in the body.

Although the linear theory of elasticity for an anisotropic medium has been known for quite a long time, a realistic solution has not been presented except in the case of orthotropic materials [1]. Solution has been presented for an anisotropic plate with one plane of elastic symmetry [2] but it involves a number of mappings into different planes and does not reach a final expression. Equations for the deflection of an isotropic thin elastic plate subjected to a temperature distribution of the form

$$T(x,y,z) = T_0(x,y) + zT_1(x,y) \tag{1}$$

have been derived by Nadai [3]. He did not, however, work on the solutions of these equations. Principally, the isothermal theory of anisotropic plates has been developed by Boussinesq [4], Voigt [5] and Lechnitzky [6]. It appears from a literature survey that until 1941 only Voigt [5] had given any consideration to the thermal effects on the anisotropic plate. He, too, considered a simple case with temperature distribution of the kind

$$T = T(x,y)$$

and, thus, did not consider any bending effect. In 1941, Sokolonikoff [7] presented a short summary of anisotropic plate theory. Pell [2] considered the general case of a plate acted upon by forces on its edges, free of body forces and having a temperature distribution as given by equation (1). After using a number of mapping functions, he arrived at a rather complicated solution.

It is the aim of this research to develop a computer program to compute the thermal deflections and stresses in an anisotropic plate that has at least one plane of elastic symmetry. The temperature distribution considered is given by equation (1). Introduction of a stress function F reduces the compatibility equation to a fourth order partial differential equation [7]. The statical equilibrium equation reduces to a fourth order partial differential equation in terms of W, the deflection. These two equations are solved on a digital computer using the finite difference technique. Once the values of F have been found, the stress distribution due to T_0 can be obtained by direct substitution of F into the corresponding stress equations. In a similar way, the values of W lead to stress distribution due to zT_1. Superposition of the latter onto the earlier ones gives the total stress distribution in the plate.

CHAPTER II

THERMOELASTIC EQUATIONS FOR ANISOTROPIC PLATES

Consider a thin elastic plate composed of a medium having at least one plane of elastic symmetry passing through each point parallel to the middle plane of the plate, which is chosen to lie in the x-y plane. Let the plate be subjected to a temperature distribution given by equation (1). The Cartesian coordinate system and the plate dimensions are shown in Figure 1.

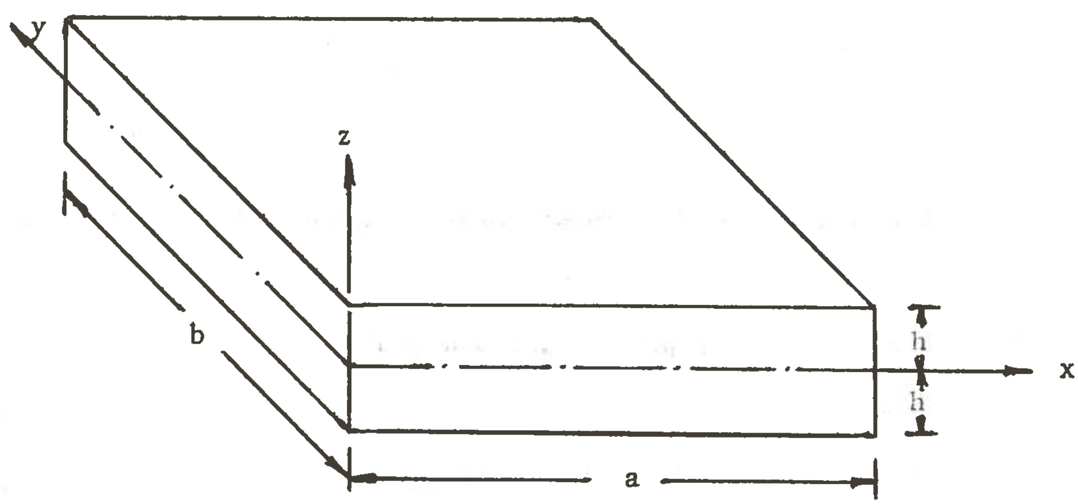

Figure 1

Coordinate System and the Plate Dimensions

The Kirchhoff assumptions [7] of the thin plate theory lead to the fundamental relations

$$u = -z\frac{\partial w}{\partial x} ,$$
$$v = -z\frac{\partial w}{\partial y} , \tag{2}$$

and

$$|\sigma_z| << \{|\sigma_x|,|\sigma_y|,|\tau_{xy}|\} , \tag{3}$$

where σ_x, σ_y, σ_z are the normal stresses in x, y and z directions, respectively, and τ_{xy} is the shear stress in the x,y plane. In these equations w is the deflection of the middle surface of the plate, and u and v are the displacements of a point (x,y,z) of the plate in the x and y directions respectively. These relations are valid throughout the thickness, 2h, of the plate. Pell [2] further assumed that the stress tensor τ at any point in the plate is the linear sum of two stress tensors:

$$\tau = \tau^0 + \tau^1 , \tag{4}$$

where τ^0 is a plane stress tensor generated by $T_0(x,y)$, and τ^1 is the bending stress tensor in the plate due to zT_1.

The generalized Hooke's law [8] connecting the components of stress at any point of an elastic solid body with the components of strain at that point, taking into account the thermal effects, is expressed by

$$\frac{\partial u}{\partial x} = a_{11}\sigma_x + a_{12}\sigma_y + a_{13}\sigma_z + a_{16}\tau_{xy} + \alpha_1 T \ ,$$

$$\frac{\partial v}{\partial y} = a_{21}\sigma_x + a_{22}\sigma_y + a_{23}\sigma_z + a_{26}\tau_{xy} + \alpha_2 T,$$

$$\frac{\partial w}{\partial z} = a_{31}\sigma_x + a_{32}\sigma_y + a_{33}\sigma_z + a_{36}\tau_{xy} + \alpha_3 T, \qquad (5)$$

$$\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = a_{44}\tau_{yz} + a_{45}\tau_{xz} + 2\alpha_4 T \ ,$$

$$\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = a_{54}\tau_{yz} + a_{55}\tau_{xz} + 2\alpha_5 T \ ,$$

and

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = a_{61}\sigma_x + a_{62}\sigma_y + a_{63}\sigma_x + a_{66}\tau_{xy} + 2\alpha_6 T \ ,$$

where the a_{ij} are elastic constants generally called compliances, and α_i are the linear coefficients of thermal expansion of the medium. Love [8] established the relationship

$$a_{ij} = a_{ji}$$

which helps reduce the number of compliances in equations (5) from 20 to 13.

For the plane stress condition, the system of equations (5) reduces to

$$\frac{\partial u^0}{\partial x} = a_{11}\sigma^0_x + a_{12}\sigma^0_y + a_{16}\tau^0_{xy} + \alpha_1 T_0 \ ,$$

$$\frac{\partial v^0}{\partial y} = a_{12}\sigma^0_x + a_{22}\sigma^0_y + a_{26}\tau^0_{xy} + \alpha_2 T_0 \ , \qquad (6)$$

and

$$\frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} = a_{16}\sigma^0_x + a_{26}\sigma^0_y + a_{66}\tau^0_{xy} + 2\alpha_6 T_0 \ .$$

In solving two-dimensional problems by means of a stress formulation, it is convenient to introduce a stress function defined in such a way that the equations of equilibrium are identically satisfied.

In such a condition, the function will then be required to satisfy the compatibility equation [9]

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = 2 \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} , \tag{7}$$

where ε_x and ε_y are unit elongations in the x and y directions and γ_{xy} is shearing strain component in the x,y plane.

In this case the stress function F is introduced in the following manner:

$$\sigma_x^0 = \frac{\partial^2 F}{\partial y^2} ; \ \sigma_y^0 = \frac{\partial^2 F}{\partial x^2} ; \ \tau_{xy}^0 = - \frac{\partial^2 F}{\partial x \partial y} . \tag{8}$$

It is further convenient in plate theory to deal with forces and moments per unit of length rather than with the stresses themselves. The following quantities are therefore introduced:

$$N_x = 2h\sigma_x^0 , \ N_y = 2h\sigma_y^0 , \ N_{xy} = 2h\tau_{xy}^0 , \tag{9}$$

where σ_x^0 , σ_y^0 , and τ_{xy}^0 are the average normal stresses in the X and Y directions and the average shearing stress over the plate thickness, respectively.

From equations (6) and (8) one would obtain

$$\varepsilon_x = a_{11} \frac{\partial^2 F}{\partial y^2} + a_{12} \frac{\partial^2 F}{\partial x^2} - a_{16} \frac{\partial^2 F}{\partial x \partial y} + \alpha_1 T_0 ,$$

$$\varepsilon_y = a_{12} \frac{\partial^2 F}{\partial y^2} + a_{22} \frac{\partial^2 F}{\partial x^2} - a_{26} \frac{\partial^2 F}{\partial x \partial y} + \alpha_2 T_0 , \tag{10}$$

and

$$\gamma_{xy} = \frac{1}{2} \{ a_{16} \frac{\partial^2 F}{\partial y^2} + a_{26} \frac{\partial^2 F}{\partial x^2} - a_{66} \frac{\partial^2 F}{\partial x \partial y} + 2\alpha_6 T_0 \} .$$

Substituting these expressions into equation (7) results in

$$a_{11} \frac{\partial^4 F}{\partial y^4} + a_{12} \frac{\partial^4 F}{\partial x^2 \partial y^2} - a_{16} \frac{\partial^4 F}{\partial x \partial y^3} + \alpha_1 \frac{\partial^2 T_0}{\partial y^2} + a_{12} \frac{\partial^4 F}{\partial x^2 \partial y^2}$$

$$+ a_{22} \frac{\partial^4 F}{\partial x^4} - a_{26} \frac{\partial^4 F}{\partial x^3 \partial y} + \alpha_2 \frac{\partial^2 T_0}{\partial x^2}$$

$$= a_{16} \frac{\partial^4 F}{\partial x \partial y^3} + a_{26} \frac{\partial^4 F}{\partial x^3 \partial y} - a_{66} \frac{\partial^4 F}{\partial x^2 \partial y^2} + 2\alpha_6 \frac{\partial^2 T_0}{\partial x \partial y} \quad .$$

Rearranging this equation leads to the relation

$$a_{22} \frac{\partial^4 F}{\partial x^4} - 2a_{26} \frac{\partial^4 F}{\partial x^3 \partial y} + (2a_{12} + a_{66}) \frac{\partial^4 F}{\partial x^2 \partial y^2} - 2a_{16} \frac{\partial^4 F}{\partial x \partial y^3} + a_{11} \frac{\partial^4 F}{\partial y^4}$$

$$= - \{ \alpha_2 \frac{\partial^2 T_0}{\partial x^2} - 2\alpha_6 \frac{\partial^2 T_0}{\partial x \partial y} + \alpha_1 \frac{\partial^2 T_0}{\partial y^2} \} \quad . \qquad (11)$$

The generalized Hooke's law [8], taking into account the thermal effects due to zT_1, is expressed by

$$\sigma_x{}^1 = c_{11}(\frac{\partial u^1}{\partial x} - \alpha_1 z T_1) + c_{12}(\frac{\partial v^1}{\partial y} - \alpha_2 z T_1)$$

$$+ c_{13}(\frac{\partial w^1}{\partial z} - \alpha_3 z T_1) + c_{16}(\frac{1}{2} [\frac{\partial u^1}{\partial y} + \frac{\partial v^1}{\partial x}] - \alpha_6 z T_1) \; ,$$

(12)

$$\sigma_y{}^1 = c_{12}(\frac{\partial u^1}{\partial x} - \alpha_1 z T_1) + c_{22}(\frac{\partial v^1}{\partial y} - \alpha_2 z T_1)$$

$$+ c_{23}(\frac{\partial w^1}{\partial z} - \alpha_3 z T_1) + c_{26}(\frac{1}{2} [\frac{\partial u^1}{\partial y} + \frac{\partial v^1}{\partial x}] - \alpha_6 z T_1) \; ,$$

$$\sigma_z^{\,1} = c_{13}\left(\frac{\partial u^1}{\partial x} - \alpha_1 z T_1 \right) + c_{23}\left(\frac{\partial v^1}{\partial y} - \alpha_2 z T_1 \right) \qquad \text{(12 cont'd)}$$

$$+ c_{33}\left(\frac{\partial w^1}{\partial z} - \alpha_3 z T_1 \right) + c_{36}\left(\frac{1}{2} \left[\frac{\partial u^1}{\partial y} + \frac{\partial v^1}{\partial x}\right] - \alpha_6 z T_1 \right) ,$$

and

$$\tau_{xy}^1 = c_{16}\left(\frac{\partial u^1}{\partial x} - \alpha_1 z T_1 \right) + c_{26}\left(\frac{\partial v^1}{\partial y} - \alpha_2 z T_1 \right)$$

$$+ c_{36}\left(\frac{\partial w^1}{\partial z} - \alpha_3 z T_1 \right) + c_{66}\left(\frac{1}{2} \left[\frac{\partial u^1}{\partial y} + \frac{\partial v^1}{\partial x}\right] - \alpha_6 z T_1 \right) .$$

In these equations the c_{ij} are elastic constants generally called stiffnesses. To obtain c_{ij} from a_{ij} and vice versa, the following formulas [10] are to be used:

$$c_{ij} = \frac{\delta_{ij}}{\Delta} ,$$

and

$$a_{ij} = \frac{\delta'_{ij}}{\Delta'} .$$

Here Δ and Δ' stand for the determinants of the square matrices $[a_{ij}]$ and $[c_{ij}]$, respectively; δ_{ij} and δ'_{ij} are the cofactors of a_{ij} and c_{ij}, respectively.

From the assumptions expressed by equations (2) and (3), the following relations result

$$\frac{\partial u^1}{\partial x} = - z \frac{\partial^2 w}{\partial x^2} ,$$

$$\frac{\partial v^1}{\partial y} = - z \frac{\partial^2 w}{\partial y^2} ,$$

$$\frac{1}{2}\left(\frac{\partial u^1}{\partial y} + \frac{\partial v^1}{\partial x}\right) = -z\frac{\partial^2 w}{\partial x \partial y} \quad ,$$

and

$$\sigma_z^1 = 0 \quad .$$

Note here that w is being written instead of w^1. This is due to the fact that T_0 does not produce any bending, thus w and w^1 are the same. Substituting these in the third equation of (12) yields

$$0 = c_{13}\left(-z\frac{\partial^2 w}{\partial x^2} - \alpha_1 z T_1\right) + c_{23}\left(-z\frac{\partial^2 w}{\partial y^2} - \alpha_2 z T_1\right)$$

$$+ c_{33}\left(\frac{\partial w}{\partial z} - \alpha_3 z T_1\right) + c_{36}\left(-z\frac{\partial^2 w}{\partial x \partial y} - \alpha_6 z T_1\right) \quad .$$

Thus

$$\frac{\partial w}{\partial z} - \alpha_3 z T_1 = \frac{z}{c_{33}}\left[c_{13}\left(\frac{\partial^2 w}{\partial x^2} + \alpha_1 T_1\right)\right.$$

$$\left. + c_{23}\left(\frac{\partial^2 w}{\partial y^2} + \alpha_2 T_1\right) + c_{36}\left(\frac{\partial^2 w}{\partial x \partial y} + \alpha_6 T_1\right)\right] \quad .$$

Substituting the expression for $(\frac{\partial w}{\partial z} - \alpha_3 z T_1)$ back into equations (12):

$$\sigma_x^1 = z[c_{11}(-\frac{\partial^2 w}{\partial x^2} - \alpha_1 T_1) + c_{12}(-\frac{\partial^2 w}{\partial y^2} - \alpha_2 T_1)$$

$$+ \frac{c_{13}}{c_{33}}\{c_{13}(\frac{\partial^2 w}{\partial x^2} + \alpha_1 T_1) + c_{23}(\frac{\partial^2 w}{\partial y^2} + \alpha_2 T_1) + c_{36}(\frac{\partial^2 w}{\partial x \partial y} + \alpha_6 T_1)\}]$$

$$= z[(\frac{c_{13}c_{13}}{c_{33}} - c_{11})\frac{\partial^2 w}{\partial x^2} + (\frac{c_{23}c_{13}}{c_{33}} - c_{12})\frac{\partial^2 w}{\partial y^2} + (\frac{c_{36}c_{13}}{c_{33}} - c_{36})\frac{\partial^2 w}{\partial x \partial y}$$

$$+ \{\alpha_1(\frac{c_{13}c_{13}}{c_{33}} - c_{11}) + \alpha_2(\frac{c_{23}c_{13}}{c_{33}} - c_{12}) + \alpha_6(\frac{c_{36}c_{13}}{c_{33}} - c_{36})\}T_1] \quad .$$

The expression is simplified by defining

$$b_{ik} = - \left(\frac{c_{i3}c_{k3}}{c_{33}} - c_{ik}\right), \qquad i,k = 1,2,6$$

and

$$\alpha_i^1 = b_{1i}\alpha_1 + b_{2i}\alpha_2 + b_{6i}\alpha_6 \quad . \qquad i = 1,2,6$$

Following the same steps for the second and fourth equations of (12), the results obtained will be

$$\sigma_x^1 = - z\left(b_{11}\frac{\partial^2 w}{\partial x^2} + b_{12}\frac{\partial^2 w}{\partial y^2} + b_{16}\frac{\partial^2 w}{\partial x \partial y} + \alpha_1^1 T_1\right) \quad ,$$

$$\sigma_y^1 = - z\left(b_{12}\frac{\partial^2 w}{\partial x^2} + b_{22}\frac{\partial^2 w}{\partial y^2} + b_{26}\frac{\partial^2 w}{\partial x \partial y} + \alpha_2^1 T_1\right) \quad , \tag{13}$$

and

$$\tau_{xy}^1 = - z\left(b_{16}\frac{\partial^2 w}{\partial x^2} + b_{26}\frac{\partial^2 w}{\partial y^2} + b_{66}\frac{\partial^2 w}{\partial x \partial y} + \alpha_6^1 T_1\right) \quad .$$

The equations of equilibrium can be expressed as

$$\frac{\partial \sigma_x^1}{\partial x} + \frac{\partial \tau_{xy}^1}{\partial y} + \frac{\partial \tau_{xz}^1}{\partial z} = 0 \quad ,$$

and

$$\frac{\partial \tau_{xy}^1}{\partial x} + \frac{\partial \sigma_y^1}{\partial y} + \frac{\partial \tau_{yz}^1}{\partial z} = 0. \tag{14}$$

From the first of these two equations

$$\frac{\partial \tau_{xz}^1}{\partial z} = - \left(\frac{\partial \sigma_x^1}{\partial x} + \frac{\partial \tau_{xy}^1}{\partial y}\right)$$

$$= z\left(b_{11}\frac{\partial^3 w}{\partial x^3} + b_{12}\frac{\partial^3 w}{\partial x \partial y^2} + b_{16}\frac{\partial^3 w}{\partial x^2 \partial y} + \alpha_1^1 \frac{\partial T_1}{\partial x}\right)$$

$$+ z\left(b_{16} \frac{\partial^3 w}{\partial x^2 \partial y} + b_{26} \frac{\partial^3 w}{\partial y^3} + b_{66} \frac{\partial^3 w}{\partial x \partial y^2} + \alpha_6^1 \frac{\partial T_1}{\partial x}\right) \quad ;$$

or simply

$$\frac{\partial \tau_{xz}^1}{\partial z} = z(A + B) \quad , \tag{15}$$

where A and B are defined as

$$A = b_{11} \frac{\partial^3 w}{\partial x^3} + b_{12} \frac{\partial^3 w}{\partial x \partial y^2} + b_{16} \frac{\partial^3 w}{\partial x^2 \partial y} + \alpha_1^1 \frac{\partial T_1}{\partial x} \quad ,$$

and

$$B = b_{16} \frac{\partial^3 w}{\partial x^2 \partial y} + b_{26} \frac{\partial^3 w}{\partial y^3} + b_{66} \frac{\partial^3 w}{\partial x \partial y^2} + \alpha_6^1 \frac{\partial T_1}{\partial y} \quad .$$

Note here that

$$\tau_{yz}^1]_{z=\pm h} = \tau_{xz}^1]_{z=\pm h} = 0 \quad .$$

If equation (15) is integrated with respect to z, then

$$\tau_{xz}^1 = \frac{z^2}{2} (A + B) + c \quad .$$

At $z = +h$,

$$0 = \frac{h^2}{2} (A + B) + c \quad ,$$

or

$$c = - \frac{h^2}{2} (A + B);$$

and thus

$$\tau_{xz}^1 = \frac{z^2 - h^2}{2} (A + B) \quad .$$

The shear force acting, per unit of length, across the thickness of the plate is

$$Q_x = \int_{-h}^{h} \tau^1_{xz} dz$$

$$= \int_{-h}^{h} \frac{z^2 - h^2}{2} (A + B)\, dz$$

$$= \frac{(A + B)}{2} \left[\frac{z^3}{3} - h^2 z \right]_{-h}^{h}$$

$$= - \frac{2}{3} h^3 (A + B)$$

Replacing A and B by their expressions in terms of w:

$$Q_x = - \frac{2}{3} h^3 \{ b_{11} \frac{\partial^3 w}{\partial x^3} + b_{12} \frac{\partial^3 w}{\partial x \partial y^2} + b_{16} \frac{\partial^3 w}{\partial x^2 \partial y} + \alpha^1_1 \frac{\partial T_1}{\partial x}$$

$$+ b_{16} \frac{\partial^3 w}{\partial x^2 \partial y} + b_{26} \frac{\partial^3 w}{\partial y^3} + b_{66} \frac{\partial^3 w}{\partial x \partial y^2} + \alpha^1_6 \frac{\partial T_1}{\partial y} \} \quad . \qquad \text{(16-a)}$$

Following the same steps for the other equation of (14) yields

$$Q_y = - \frac{2}{3} h^3 \{ b_{16} \frac{\partial^3 w}{\partial x^3} + b_{26} \frac{\partial^3 w}{\partial x \partial y^2} + b_{12} \frac{\partial^3 w}{\partial x^2 \partial y} + \alpha^1_6 \frac{\partial T_1}{\partial x}$$

$$+ b_{66} \frac{\partial^3 w}{\partial x^2 \partial y} + b_{22} \frac{\partial^3 w}{\partial y^3} + b_{26} \frac{\partial^3 w}{\partial x \partial y^2} + \alpha^1_2 \frac{\partial T_1}{\partial y} \} \qquad \text{(16-b)}$$

The condition of statical equilibrium [9] of an arbitrary volume element of the plate is represented by the equation

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} = 0 \quad .$$

Substituting the expressions for Q_x and Q_y yields

$$- \frac{2}{3} h^3 \{ b_{11} \frac{\partial^4 w}{\partial x^4} + 2b_{16} \frac{\partial^4 w}{\partial x^3 \partial y} + (b_{12} + b_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + b_{26} \frac{\partial^4 w}{\partial x \partial y^3}$$

$$+ \alpha_1^1 \frac{\partial^2 T_1}{\partial x^2} + \alpha_6^1 \frac{\partial^2 T_1}{\partial x \partial y} + b_{16} \frac{\partial^4 w}{\partial x^3 \partial y} + (b_{12} + b_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2}$$

$$+ 2b_{26} \frac{\partial^4 w}{\partial x \partial y^3} + b_{22} \frac{\partial^4 w}{\partial y^4} + \alpha_6^1 \frac{\partial^2 T_1}{\partial x \partial y} + \alpha_2^1 \frac{\partial^2 T_1}{\partial y^2} \}$$

$$+ N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} = 0 \quad .$$

Rearranging of the above equation leads to a fourth order partial differential equation in w

$$b_{11} \frac{\partial^4 w}{\partial x^4} + 3b_{16} \frac{\partial^4 w}{\partial x^3 \partial y} + 2(b_{12} + b_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + 3b_{26} \frac{\partial^4 w}{\partial x \partial y^3} + b_{22} \frac{\partial^4 w}{\partial y^4}$$

$$= - \{ \alpha_1^1 \frac{\partial^2 T_1}{\partial x^2} + 2\alpha_6^1 \frac{\partial^2 T_1}{\partial x \partial y} + \alpha_2^1 \frac{\partial^2 T_1}{\partial y^2} \}$$

$$+ \frac{3}{2h^3} \{ N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \} \quad . \qquad (17)$$

CHAPTER III

SOLUTION APPROACH

The thermoelastic problem described in the previous chapter is solved on a digital computer. The finite difference method is used to solve the equations (11) and (17) for the stress function F and deflection W, respectively. The computed values of F go into the equations (8) to furnish the stress tensor τ^0. Also, the computed values of W, when substituted into the equations (13), give rise to stress tensor τ^1. To help explain the method, consider a plate superimposed by a grid or lattice with a mesh size of Δh by Δk as shown in Figure 2.

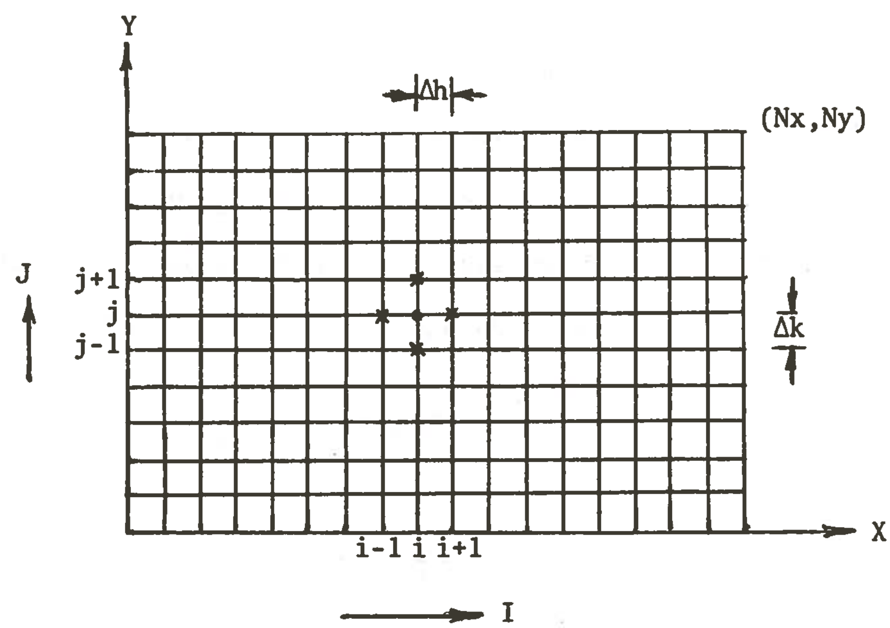

Figure 2

A Grid Superimposed on a Plate

In the case of equation (11) the numerical solution will consist of determining the values of the function F at the finitely spaced grid points shown. In this process, a combination of first, second, third, and fourth partial derivatives of F with respect to X, Y, and X and Y will be required at the grid points. These partial derivatives are given in Table A-1 [11] of Appendix A. The following paragraphs briefly describe the method for obtaining them.

The Taylor series for the function F = F(x) at points (I+1) and (I-1) expanded about point (I) gives:

$$F_{i+1} = F_i + F_i' \Delta h + F_i'' \frac{(\Delta h)^2}{2} + F_i''' \frac{(\Delta h)^3}{6} ,$$

and (18)

$$F_{i-1} = F_i - F_i' \Delta h + F_i'' \frac{(\Delta h)^2}{2} - F_i''' \frac{(\Delta h)^3}{6} ,$$

where F_i', F_i'', and F_i''' are first, second, and third partial derivatives of F_i with respect to X, respectively.

For obtaining the expressions for F_i' and F_i'', terms containing F_i''' and higher order partial derivatives may be neglected. Subtracting one equation from the other one obtains:

$$F_{i+1} - F_{i-1} = 2 F_i' \Delta h$$

or

$$\Delta h \frac{\partial F}{\partial x} = \Delta h F_i' = \frac{1}{2} \{ F_{i+1} - F_{i-1} \}$$

This is a central difference equation for the first partial derivative of F with respect to X using only three grid points.

Next, adding the two equations of (18) results in

$$F_{i-1} + F_{i+1} = 2\, F_i + F_i''(\Delta h)^2 \quad ,$$

or

$$(\Delta h)^2 \frac{\partial^2 F}{\partial x^2} = (\Delta h)^2 F_i'' = F_{i-1} - 2F_i + F_{i+1}$$

This is a central difference equation for the second partial derivative of F with respect to X using only three grid points.

Similar expressions are obtained for third and fourth partial derivatives using more grid points.

The appropriate expressions for partial derivatives are substituted into the partial differential equation, and the resulting equation is solved at various grid points for the values of required function.

Finite Difference Equation for F

Referring to Table A-1, it may be noted that the partial derivatives required for solving equation (11) can be written as

$$\frac{\partial^4 F}{\partial x^4} = \frac{1}{(\Delta h)^4}\{ F_{i-2,j} - 4F_{i-1,j} + 6F_{i,j} - 4F_{i+1,j} + F_{i+2,j}\} \quad ,$$

$$\frac{\partial^4 F}{\partial x^3 \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial^3 F}{\partial x^3}\right)$$

$$= \frac{\partial}{\partial y}\left[\frac{1}{2(\Delta h)^3}\{-F_{i-2,j} + 2F_{i-1,j} - 2F_{i+1,j} + F_{i+2,j}\}\right] \qquad (19)$$

$$= \frac{1}{24(\Delta h)^3 \Delta k}[-\{F_{i-2,j-2} - 8F_{i-2,j-1} + 8F_{i-2,j+1} - F_{i-2,j+2}\}$$

$$+ 2\{F_{i-1,j-2} - 8F_{i-1,j-1} + 8F_{i-1,j+1} - F_{i-1,j+2}\}$$

$$- 2\{F_{i+1,j-2} - 8F_{i+1,j-1} + 8F_{i+1,j+1} - F_{i+1,j+2}\}$$

$$+ \{F_{i+2,j-2} - 8F_{i+2,j-1} + 8F_{i+2,j+1} - F_{i+2,j+2}\}] \quad ,$$

$$\frac{\partial^4 F}{\partial x^2 \partial y^2} = \frac{1}{144(\Delta h)^2(\Delta k)^2} [-\{-F_{i-2,j-2} + 16F_{i-2,j-1} - 30F_{i-2,j}$$

$$+ 16F_{i-2,j+1} - F_{i-2,j+2}\} + 16\{- F_{i-1,j-2} + 16F_{i-1,j-1}$$

$$- 30F_{i-1,j} + 16F_{i-1,j+1} - F_{i-1,j+2}\} - 30\{-F_{i,j-2}$$

$$+ 16F_{i,j-1} - 30F_{i,j} + 16F_{i,j+1} - F_{i,j+2}\}$$

$$+ 16\{- F_{i+1,j-2} + 16F_{i+1,j-1} - 30F_{i+1,j} + 16F_{i+1,j+1}$$

$$- F_{i+1,j+2}\} - \{-F_{i+2,j-2} + 16F_{i+2,j-1} - 30F_{i+2,j}$$

$$+ 16F_{i+2,j+1} - F_{i+2,j+2}\}] \quad ,$$

(19) cont'd

$$\frac{\partial^4 F}{\partial x \partial y^3} = \frac{1}{24\Delta h(\Delta k)^3} [\{-F_{i-2,j-2} + 2F_{i-2,j-1} - 2F_{i-2,j+1} + F_{i-2,j+2}\}$$

$$- 8\{-F_{i-1,j-2} + 2F_{i-1,j-1} - 2F_{i-1,j+1} + F_{i-1,j+2}\}$$

$$+ 8\{-F_{i+1,j-2} + 2F_{i+1,j-1} - 2F_{i+1,j+1} + F_{i+1,j+2}\}$$

$$- \{-F_{i+2,j-2} + 2F_{i+2,j-1} - 2F_{i+2,j+1} + F_{i+2,j+2}\}] \quad ,$$

$$\frac{\partial^4 F}{\partial y^4} = \frac{1}{(\Delta k)^4} \{F_{i,j-2} - 4F_{i,j-1} + 6F_{i,j} - 4F_{i,j+1} + F_{i,j+2}\} \quad .$$

The constants P, Q, R, S, and T are defined such that:

$$P = \frac{a_{22}}{(\Delta h)^4} \quad , \qquad Q = \frac{a_{26}}{12(\Delta h)^3(\Delta k)}$$

(20)

$$R = \frac{(2a_{12} + a_{66})}{144(\Delta h)^2(\Delta k)^2} \quad , \qquad S = \frac{a_{16}}{12\Delta h(\Delta k)^3} \quad ,$$

and

$$T = \frac{a_{11}}{(\Delta k)^4} \quad . \qquad (20) \text{ cont'd}$$

Next, the right-hand side of equation (11) can be represented by a function RHSF (x,y) for a digital computer so that

$$\text{RHSF } (x,y) = \alpha_2 \frac{\partial^2 T_0}{\partial x^2} - 2\alpha_6 \frac{\partial^2 T_0}{\partial x \partial y} + \alpha_1 \frac{\partial^2 T_0}{\partial y^2} \quad .$$

Substituting the expressions (19), the constants defined by equations (20), and the function RHSF (x,y) into the equation (11), and then rearranging the resulting equation, one would obtain the expression for $F_{i,j}$, the value of F at any point (I,J), as

$$\begin{aligned} F_{i,j} = - [&BM_1 F_{i-2,j-2} + BM_2 F_{i-2,j-1} + BM_3 F_{i-2,j} + BM_4 F_{i-2,j+1} \\ &+ BM_5 F_{i-2,j+2} + BM_6 F_{i-1,j-2} + BM_7 F_{i-1,j-1} + BM_8 F_{i-1,j} \\ &+ BM_9 F_{i-1,j+1} + BM_{10} F_{i-1,j+2} + BM_{11} F_{i,j-2} + BM_{12} F_{i,j-1} \\ &+ BM_{14} F_{i,j+1} + BM_{15} F_{i,j+2} + BM_{16} F_{i+1,j-2} + BM_{17} F_{i+1,j-1} \\ &+ BM_{18} F_{i+1,j} + BM_{19} F_{i+1,j+1} + BM_{20} F_{i+1,j+2} + BM_{21} F_{i+2,j-2} \\ &+ BM_{22} F_{i+2,j-1} + BM_{23} F_{i+2,j} + BM_{24} F_{i+2,j+1} + BM_{25} F_{i+2,j+2} \\ &+ \text{RHSF } (x,y)] \, / BM_{13} \quad . \end{aligned} \qquad (21)$$

In the above equation, the coefficients BM_i are defined as

$$BM_1 = BM_{25} = Q + R + S \quad ,$$
$$BM_2 = BM_{24} = -8Q - 16R - 2S \quad ,$$
$$BM_3 = BM_{23} = P + 30R \quad ,$$
$$BM_4 = BM_{22} = 8Q - 16R + 2S \quad ,$$
$$BM_5 = BM_{21} = -Q + R - S \quad ,$$
$$BM_6 = BM_{20} = -2Q - 16R - 8S \quad , \qquad (22)$$
$$BM_7 = BM_{19} = 16Q + 256R + 16S \quad ,$$
$$BM_8 = BM_{18} = -4P - 480R \quad ,$$
$$BM_9 = BM_{17} = -16Q + 256R - 16S \quad ,$$
$$BM_{10} = BM_{16} = 2Q - 16R + 8S \quad ,$$
$$BM_{11} = BM_{15} = 30R + T \quad ,$$
$$BM_{12} = BM_{14} = -480R - 4T \quad ,$$

and

$$BM_{13} = 6P + 900R + 6T \quad .$$

Finite Difference Equation for W

Due to the similarity between the left hand sides of the two equations (11) and (17), the left hand side of the finite difference form of equation (17) for W will be similar to the one for F, the only major change being in the coefficients.

Constants P1, Q1, R1, S1, and T1 are defined as

$$P1 = \frac{b_{11}}{(\Delta h)^4} \quad , \qquad Q1 = -\frac{3b_{16}}{24(\Delta h)^3 \Delta k} \quad ,$$

$$R1 = \frac{2(b_{12} + b_{66})}{144(\Delta h)^2(\Delta k)^2} \quad , \qquad S1 = -\frac{3b_{26}}{24\Delta h(\Delta k)^3} \quad , \qquad (23)$$

and

$$T1 = \frac{b_{22}}{(\Delta k)^4} \quad .$$

Thus the left hand side, LHS, of the equation (17) can be written as

$$\begin{aligned} LHS = {} & DM_1 W_{i-2,j-2} + DM_2 W_{i-2,j-1} + DM_3 W_{i-2,j} + DM_4 W_{i-2,j+1} \\ & + DM_5 W_{i-2,j+2} + DM_6 W_{i-1,j-2} + DM_7 W_{i-1,j-1} + DM_8 W_{i-1,j} \\ & + DM_9 W_{i-1,j+1} + DM_{10} W_{i-1,j+2} + DM_{11} W_{i,j-2} + DM_{12} W_{i,j-1} \\ & + DM_{13} W_{i,j} + DM_{14} W_{i,j+1} + DM_{15} W_{i,j+2} + DM_{16} W_{i+1,j-2} \\ & + DM_{17} W_{i+1,j-1} + DM_{18} W_{i+1,j} + DM_{19} W_{i+1,j+1} + DM_{20} W_{i+1,j+2} \\ & + DM_{21} W_{i+2,j-2} + DM_{22} W_{i+2,j-1} + DM_{23} W_{i+2,j} + DM_{24} W_{i+2,j+1} \\ & + DM_{25} W_{i+2,j+2} \quad , \end{aligned} \tag{24}$$

where the coefficients DM_i are expressed in terms of P1, Q1, R1, S1, and T1 as

$$\begin{aligned} DM_1 &= DM_{25} = Q1 + R1 + S1 \quad , \\ DM_2 &= DM_{24} = -\ 8Q1 - 16R1 - 2S1 \quad , \\ DM_3 &= DM_{23} = P1 + 30R1 \quad , \\ DM_4 &= DM_{22} = 8Q1 - 16R1 + 2S1 \quad , \\ DM_5 &= DM_{21} = -\ Q1 + R1 - S1 \quad , \\ DM_6 &= DM_{20} = -\ 2Q1 - 16R1 - 8S1 \quad , \\ DM_7 &= DM_{19} = 16Q1 + 256R1 + 16S1 \quad , \\ DM_8 &= DM_{18} = -\ 4P1 - 480R1 \quad , \\ DM_9 &= DM_{17} = -\ 16Q1 + 256R1 - 16S1 \quad , \\ DM_{10} &= DM_{16} = 2Q1 - 16R1 + 8S1 \quad , \\ DM_{11} &= DM_{15} = 30R1 + T1 \quad , \\ DM_{12} &= DM_{14} = -\ 480R1 - 4T1 \quad , \end{aligned} \tag{25}$$

and

$$DM_{13} = 6P1 + 900R1 + 6T1 \quad .$$

The right hand side of the equation (17) can be represented as

$$RHS = - RHSW\ (x,y) + \frac{3}{2h^3} \{N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2}\} \quad , \qquad (26)$$

where

$$RHSW\ (x,y) = \alpha_1^1 \frac{\partial^2 T_1}{\partial x^2} + 2\alpha_6^1 \frac{\partial^2 T_1}{\partial x \partial y} + \alpha_2^1 \frac{\partial^2 T_1}{\partial y^2} \quad .$$

Again, looking at Table A-1 in Appendix A one can write

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{(\Delta h)^2} \{W_{i-1,j} - 2W_{i,j} + W_{i+1,j}\}$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{1}{(\Delta k)^2} \{W_{i,j-1} - 2W_{i,j} + W_{i,j+1}\} \qquad (27)$$

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{1}{4\Delta h \Delta k} \{W_{i-1,j-1} - W_{i-1,j+1} - W_{i+1,j-1} + W_{i+1,j+1}\}$$

Substituting the expressions for LHS and RHS into the equation (17) yields

$$\begin{aligned} W_{i,j} = - [&DM_1 W_{i-2,j-2} + DM_2 W_{i-2,j-1} + DM_3 W_{i-2,j} + DM_4 W_{i-2,j+1} \\ &+ DM_5 W_{i-2,j+2} + DM_6 W_{i-1,j-2} + DM_7 W_{i-1,j-1} + \{DM_8 \\ &- h_1 N_{x_{i,j}} /(\Delta h)^2\} W_{i-1,j} + DM_9 W_{i-1,j+1} + DM_{10} W_{i-1,j+2} \\ &+ DM_{11} W_{i,j-2} + \{DM_{12} - h_1 N_{y_{i,j}} /(\Delta k)^2\} W_{i,j-1} \\ &+ \{DM_{14} - h_1 N_{y_{i,j}} /(\Delta k)^2\} W_{i,j+1} + DM_{15} W_{i,j+2} \end{aligned} \qquad (28)$$

$$+ DM_{16}W_{i+1,j-2} + DM_{17}W_{i+1,j-1} + \{DM_{18} - h_1 N_{x_{i,j}} /$$

$$(\Delta h)^2\} W_{i+1,j} + DM_{19}W_{i+1,j+1} + DM_{20}W_{i+1,j+2}$$

$$+ DM_{21}W_{i+2,j-2} + DM_{22}W_{i+2,j-1} + DM_{23}W_{i+2,j} \qquad (28)\text{ cont'd}$$

$$+ DM_{24}W_{i+2,j+1} + DM_{25}W_{i+2,j+2} - h_1 N_{xy_{i,j}}$$

$$\{W_{i-1,j-1} - W_{i-1,j+1} - W_{i+1,j-1} + W_{i+1,j+1}\} /$$

$$(2\Delta h \Delta k) + RHSW(x,y)] \;/\; [DM_{13} + 2h_1 \{ N_{x_{i,j}} /(\Delta h)^2$$

$$+ N_{y_{i,j}} /(\Delta k)^2\}] \quad ,$$

where

$$h_1 = 3/2h^3 \quad .$$

Boundary Conditions--All Edges Free

The problem described here is a boundary value problem; that is, the state of stress and/or deflection is defined at all four edges of the plate. The following are mathematical representations of the boundary conditions in terms of F and W.

A. Boundary Conditions in Terms of F

From the free edge condition, it follows that the normal forces are zero. Also, noting that there is no bending due to T_0, it is evident that the shear force is zero. Mathematically [2] the boundary conditions on the edge X = 0,a are

$$N_x = N_{xy} = 0 \quad .$$

This leads to the relations [9]

$$F = \frac{\partial F}{\partial x} = 0 \quad ;$$

and on the edges $Y = 0,b$,

$$F = \frac{\partial F}{\partial y} = 0 \quad .$$

B. Boundary Conditions in Terms of W

At the plate's free edge the bending moment M_n, the shear force Q_n, and the twisting moment M_{ns} must all vanish, where (n,s) represents an orthogonal coordinate system, n being normal, and s being parallel to the edge. The twisting moment is, however, statically equivalent to a distributed shear force of intensity $[-(\partial M_{ns}/\partial s)_{n=0}]$.

Mathematically the boundary conditions are

$$M_n = 0 \quad ,$$

and

$$Q_n - \frac{\partial M_{ns}}{\partial s} = 0 \qquad (29)$$

To obtain Q_n in terms of moments, consider the forces in the Z direction and moments about X and Y axis acting on an element of volume dxdydz of the plate as shown in Figure 3. The moments M_x, M_y, and M_{xy} are given as

$$M_x = \int_{-h}^{h} \sigma_x z dz \quad ,$$

$$M_y = \int_{-h}^{h} \sigma_y z dz \quad , \qquad (30)$$

and

$$M_{xy} = \int_{-h}^{h} \tau_{xy} z dz \quad ;$$

and the shear forces Q_x and Q_y are represented by the equations (16-A) and (16-B).

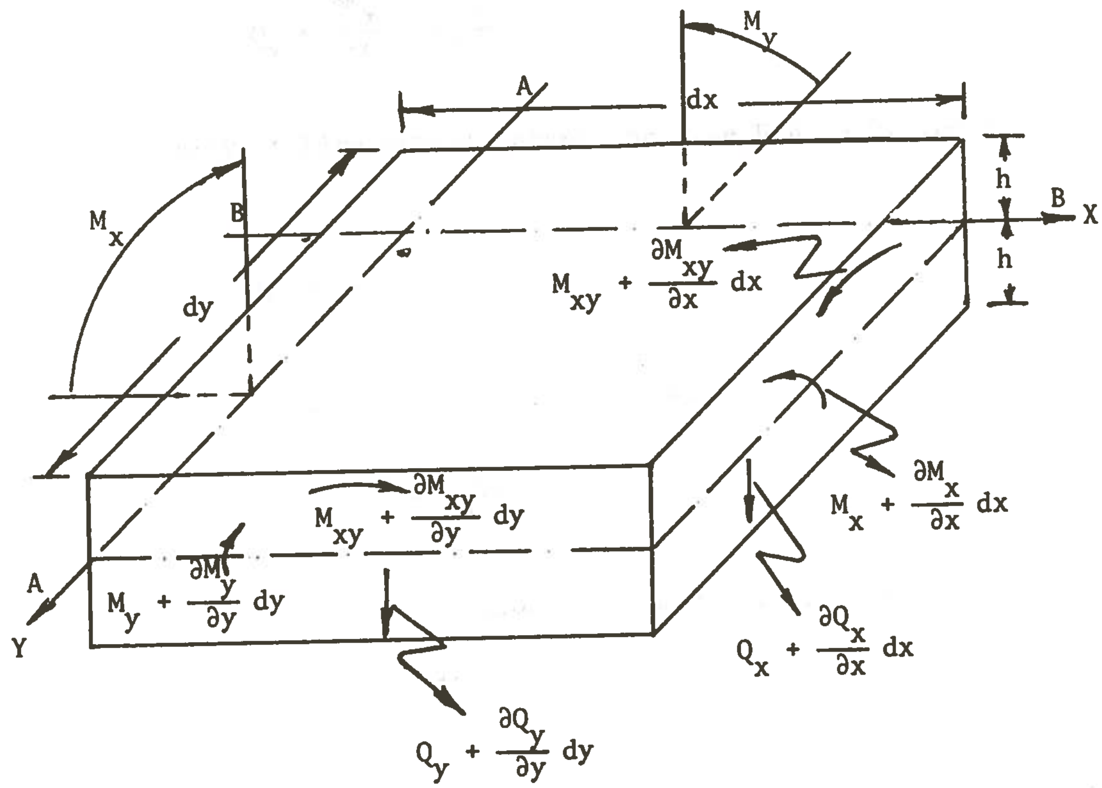

Figure 3

Moments and Shear Forces Acting on a Volume Element

Taking moments about the line A-A in Figure 3 one obtains

$$\left(M_x + \frac{\partial M_x}{\partial x} dx\right) dy - M_x dy + M_{xy} dx - \left(M_{xy} + \frac{\partial M_{xy}}{\partial y} dy\right) dx$$

$$- \left(Q_x + \frac{\partial Q_x}{\partial x} dx\right) dydx + Q_y dx \frac{dx}{2} - \left(Q_y + \frac{\partial Q_y}{\partial y} dy\right) dx \frac{dx}{2} = 0 \quad .$$

By neglecting the terms containing the product dxdydz, the above equation becomes

$$Q_x = \frac{\partial M_x}{\partial x} - \frac{\partial M_{xy}}{\partial y} \quad .$$

Similarly, taking moments about the line B-B in Figure 3 results in

$$Q_y = \frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x} \quad .$$

Replacing x and y by n and s, the shear force equation becomes

$$Q_n = \frac{\partial M_n}{\partial n} - \frac{\partial M_{ns}}{\partial s} \quad . \tag{31}$$

Interaction of the equations (29) and (31) results in the following boundary conditions:

$$M_n = 0 \quad ,$$

and (32)

$$\frac{\partial M_n}{\partial n} - 2\,\frac{\partial M_{ns}}{\partial s} = 0 \quad .$$

<u>On the Edges X = 0,a</u>

From the equations (13) and (30)

$$M_x = -\int_{-h}^{h} z^2 \left(b_{11} \frac{\partial^2 w}{\partial x^2} + b_{12} \frac{\partial^2 w}{\partial y^2} + b_{16} \frac{\partial^2 w}{\partial x \partial y} + \alpha_1^1 T_1\right)$$

$$= -\frac{2}{3} h^3 \left(b_{11} \frac{\partial^2 w}{\partial x^2} + b_{12} \frac{\partial^2 w}{\partial y^2} + b_{16} \frac{\partial^2 w}{\partial x \partial y} + \alpha_1^1 T_1\right) \quad .$$

Similarly, M_y and M_{xy} are

$$M_y = -\frac{2}{3} h^3 (b_{12} \frac{\partial^2 w}{\partial x^2} + b_{22} \frac{\partial^2 w}{\partial y^2} + b_{26} \frac{\partial^2 w}{\partial x \partial y} + \alpha_2^1 T_1) \quad ,$$

and

$$M_{xy} = -\frac{2}{3} h^3 (b_{16} \frac{\partial^2 w}{\partial x^2} + b_{26} \frac{\partial^2 w}{\partial y^2} + b_{66} \frac{\partial^2 w}{\partial x \partial y} + \alpha_6^1 T_1) \quad .$$

Further, differentiating M_x and M_{xy}:

$$\frac{\partial M_x}{\partial x} - 2 \frac{\partial M_{xy}}{\partial y} = -\frac{2}{3} h^3 (b_{11} \frac{\partial^3 w}{\partial x^3} + b_{12} \frac{\partial^3 w}{\partial x \partial y^2} + b_{16} \frac{\partial^3 w}{\partial x^2 \partial y}$$

$$+ \alpha_1^1 \frac{\partial T_1}{\partial x}) + \frac{4}{3} h^3 (b_{16} \frac{\partial^3 w}{\partial y \partial x^2} + b_{26} \frac{\partial^3 w}{\partial y^3}$$

$$+ b_{66} \frac{\partial^3 w}{\partial x \partial y^2} + \alpha_6^1 \frac{\partial T_1}{\partial y})$$

$$= -\frac{2}{3} h^3 (b_{11} \frac{\partial^3 w}{\partial x^3} - b_{16} \frac{\partial^3 w}{\partial x^2 \partial y} + (b_{12} - 2b_{66}) \frac{\partial^3 w}{\partial x \partial y^2}$$

$$- 2b_{26} \frac{\partial^3 w}{\partial y^3} + \alpha_1^1 \frac{\partial T_1}{\partial x} - 2\alpha_6^1 \frac{\partial T_1}{\partial y}) \quad .$$

Thus the equations (32) reduce to

$$b_{11} \frac{\partial^2 w}{\partial x^2} + b_{12} \frac{\partial^2 w}{\partial y^2} + b_{16} \frac{\partial^2 w}{\partial x \partial y} + \alpha_1^1 T_1 = 0 \quad ,$$

and

$$b_{11} \frac{\partial^3 w}{\partial x^3} - b_{16} \frac{\partial^3 w}{\partial x^2 \partial y} + (b_{12} - 2b_{66}) \frac{\partial^3 w}{\partial x \partial y^2} - 2b_{26} \frac{\partial^3 w}{\partial y^3} \qquad (33)$$

$$+ \alpha_1^1 \frac{\partial T_1}{\partial x} - 2\alpha_6^1 \frac{\partial T_1}{\partial y} = 0 \quad .$$

On the Edges Y = 0,b

Proceeding similarly to obtain the boundary conditions in terms of W for the edges normal to Y-axis, the following equations are to be satisfied on the edges:

$$b_{12} \frac{\partial^2 w}{\partial x^2} + b_{22} \frac{\partial^2 w}{\partial y^2} + b_{26} \frac{\partial^2 w}{\partial x \partial y} + \alpha_2^1 T_1 = 0 \quad ,$$

and

$$2b_{16} \frac{\partial^3 w}{\partial x^3} + (2b_{66} - b_{12}) \frac{\partial^3 w}{\partial x^2 \partial y} + b_{26} \frac{\partial^3 w}{\partial x \partial y^2} \tag{34}$$

$$- b_{22} \frac{\partial^3 w}{\partial y^3} + 2\alpha_6^1 \frac{\partial T_1}{\partial x} - \alpha_2^1 \frac{\partial T_1}{\partial y} = 0 \quad .$$

CHAPTER IV

THE COMPUTER PROGRAM

The computer program is written in Fortran IV language for the IBM System 360, Model 65 computer. It is designed to compute the thermal stresses and deflections in an anisotropic plate, having at least one plane of elastic symmetry, being free of external forces and having all four edges free. A flow chart for the main program is shown in Figure 4. The flow charts for the subroutines written specifically to solve the stress analysis problem are shown in Figures 5 to 8 inclusive. The temperature distributions that are to be read into the computer are represented by fourth order polynomials in X, and Y, namely,

$$T_0 = u_1x^4 + u_2x^3y + u_3x^3 + u_4x^2y^2 + u_5x^2y + u_6x^2 + u_7x + u_8xy + u_9y + u_{10}y^2 + u_{11}xy^2 + u_{12}y^3 + u_{13}xy^3 + u_{14}y^4 + u_{15},$$

and

$$T_1 = v_1x^4 + v_2x^3y + v_3x^3 + v_4x^2y^2 + v_5x^2y + v_6x^2 + v_7x + v_8xy + v_9y + v_{10}y^2 + v_{11}xy^2 + v_{12}y^3 + v_{13}xy^3 + v_{14}y^4 + v_{15},$$

(35)

where u_1, u_2, . . . u_{15}, and v_1, v_2, . . . v_{15} are coefficients of the polynomials.

Data is read into the computer in the following sequence:

CARD 1

Nx, Ny (the number of grid points along x, and y directions of the plate, respectively), Δh, Δk (the increments along x, and y directions, respectively), and h, the half thickness of the plate.

CARDS 2 to 7

Elastic constants or compliances a_{ij} (there will be 36 of them).

CARD 8

The linear coefficients of thermal expansion in the i th direction, α_i.

CARDS 9 to 12

The coefficients for the polynomials given by (35). These are u_1, u_2, . . . u_{15}, and v_1, v_2, . . . v_{15}.

CARDS 13 to 16

Repeat coefficients from cards 9 to 12. This is done to facilitate data transfer within the program.

The entire program with sample input is listed in the Appendix A.

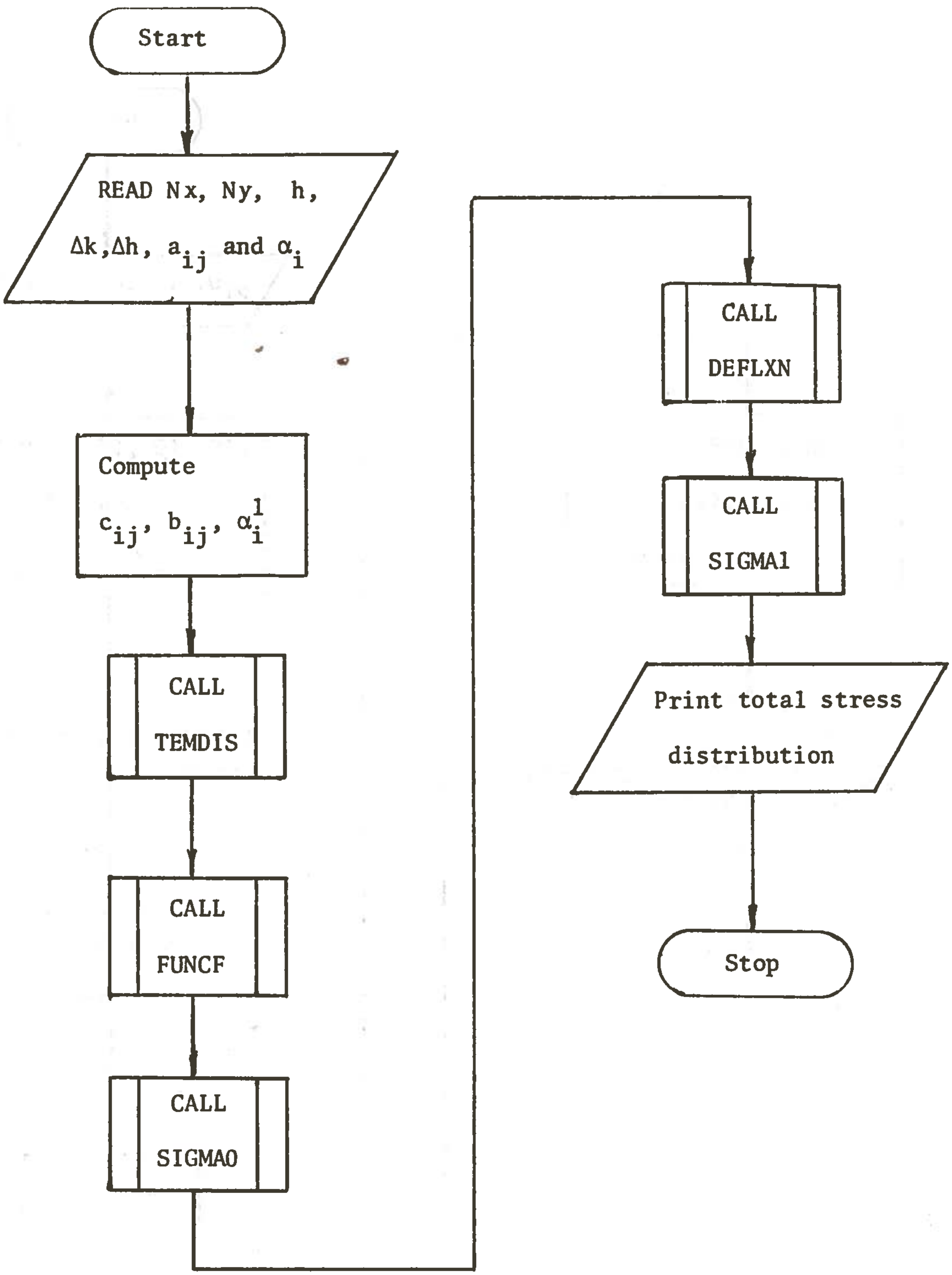

Figure 4

Main Program Flow Chart

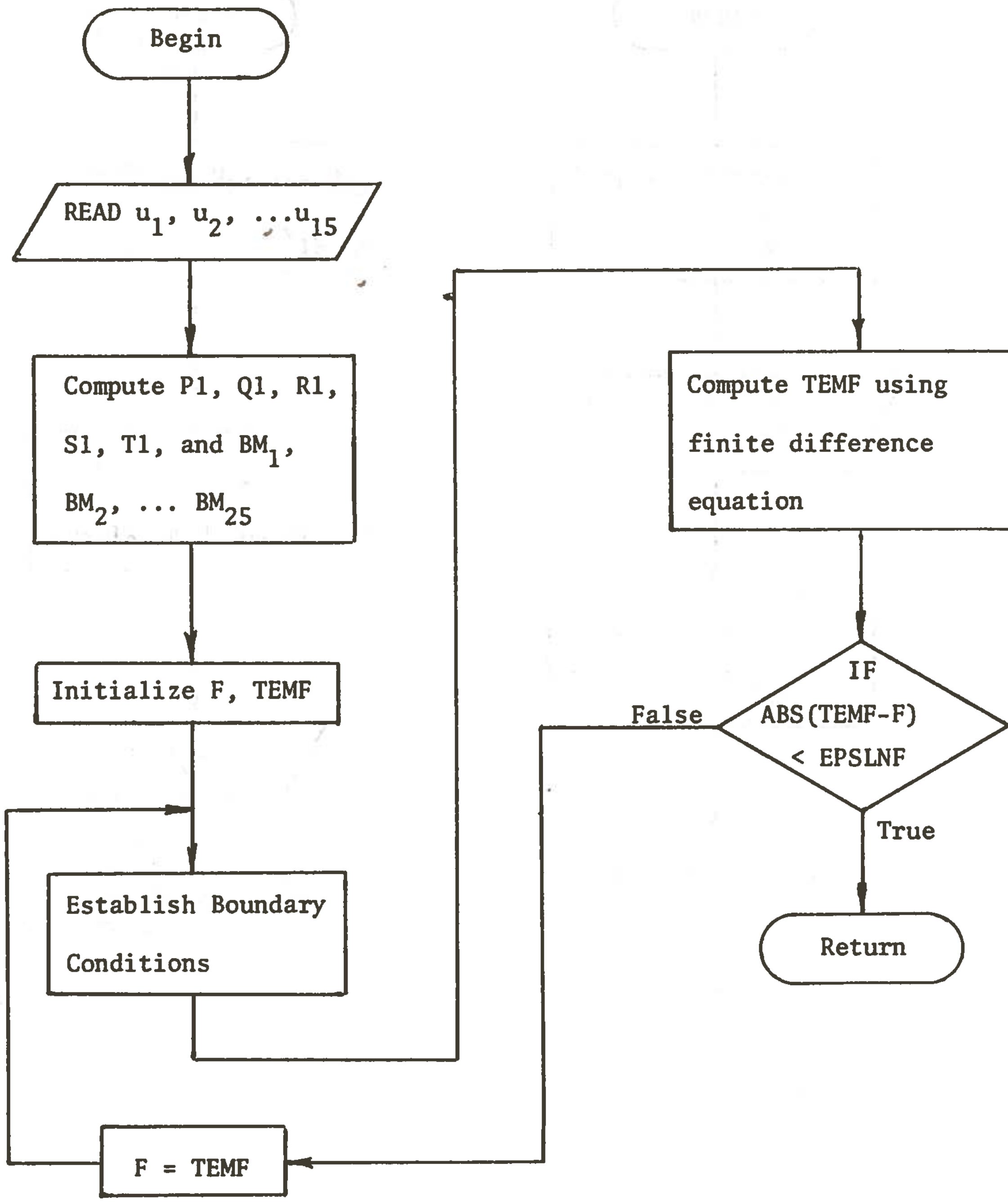

Figure 5

Subroutine FUNCF Flow Chart

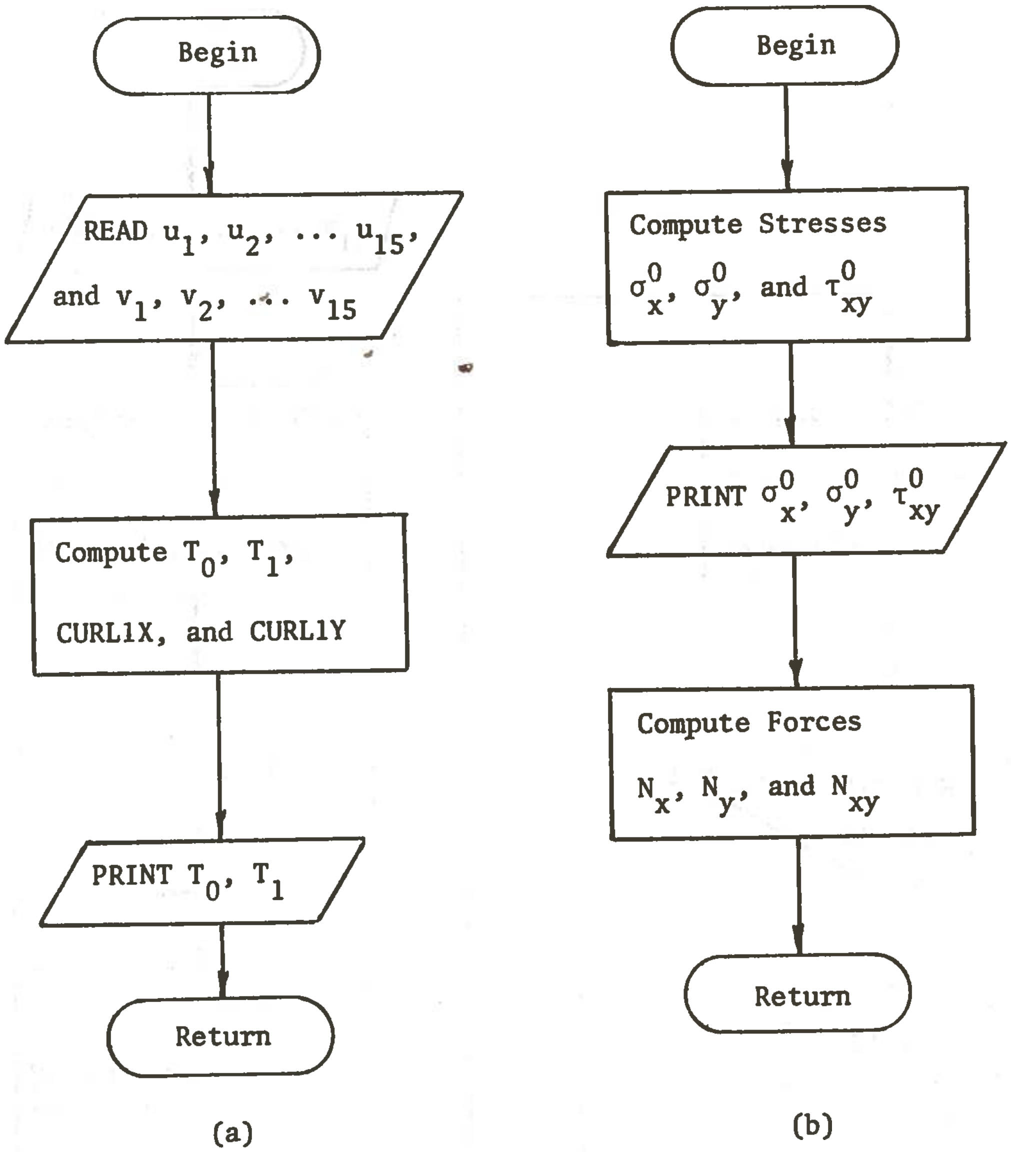

Figure 6

Flow Charts for Subroutines TEMDIS (6a) and SIGMA0 (6b)

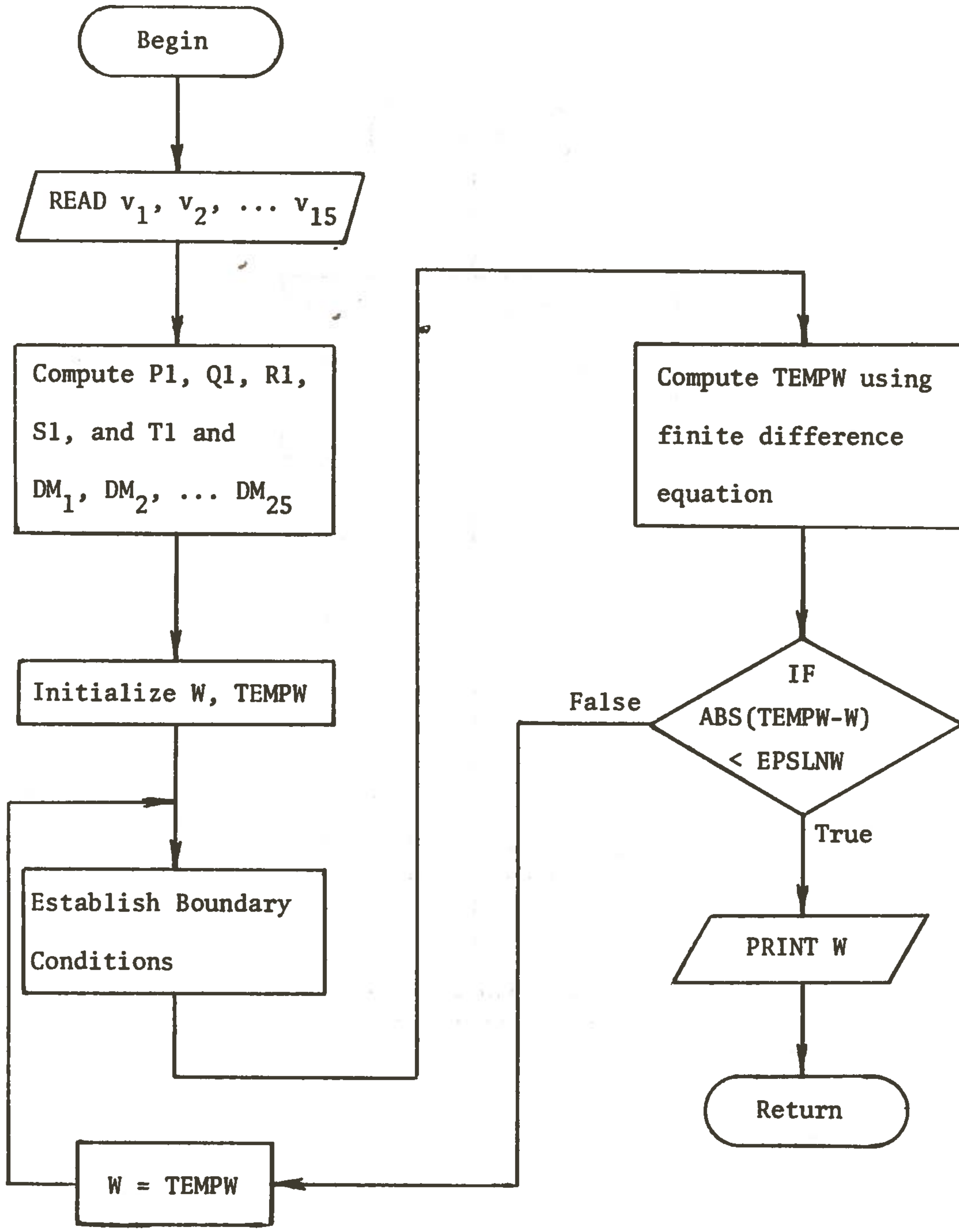

Figure 7

Subroutine DEFLXN Flow Chart

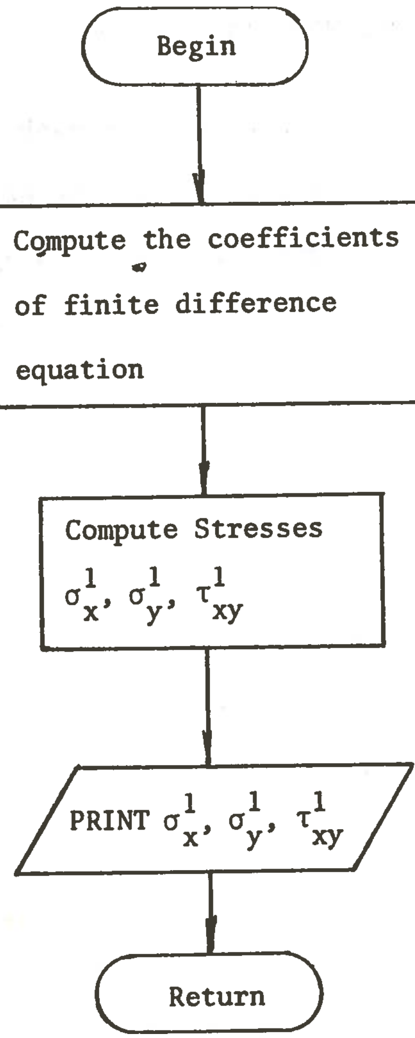

Figure 8

Subroutine SIGMA1 Flow Chart

RESULTS AND CONCLUSIONS

The program developed is capable of computing thermal stresses and deflections in an anisotropic plate having one plane of elastic symmetry and no edge constraints. Results for a sample problem (input-data included in Appendix A) are given in Appendix B. The results are not completely satisfactory. This is due to the fact that the elastic constants, a_{ij}, are not exact. These constants are not available in the literature. These are known for a number of crystalline materials but, unfortunately, they are not of interest. Thus the elastic constants were assumed to be similar to those of steel which is an isotropic material. The program was run on a trial and error basis, changing some of these elastic constants on every run. The results given in Appendix B seem to be reasonable for structural materials.

It is suggested that further work in this field be directed toward experimentally obtaining the elastic constants of interest, so that they may be analyzed for stresses and deflections due to temperature gradients. The program itself could be modified by attempting to solve the set of linear finite difference equations implicitly.

BIBLIOGRAPHY

1. Ambartsumian, S. A. "On the Theory of Bending of Anisotropic Plates and Shallow Shells," PMM, Vol. 24, No. 2, 1960.

2. Pell, William H. "Thermal Deflections of Anisotropic Thin Plates," Quarterly of Applied Mathematics, Vol. IV, No. 1, 1946.

3. Nadai, A. Elastische Platten. Springer, Berlin, 1925.

4. Boussinesq, M. J. J. de Math. (Liousville) (3) 5, 1879.

5. Voigt, W. Lehrbuch der kristallphysik. Teubner, Leipzig and Berlin, 1910.

6. Lechnitzky, S. G. App. Math. Mech. Leningrad, 3, old series 1, (1936), new series 1, (1939), new series 2, (1938).

7. Sokolonikoff, I. S. Mathematical Theory of Elasticity. Brown University, Providence, R.I., 1941.

8. Love, A. E. H. A Treatise on the Mathematical Theory of Elasticity, 1934.

9. Boley, Bruno A. and Weiner, Jerome H. Theory of Thermal Stresses. John Wiley and Sons, Inc., New York, 1960.

10. Krishnan, R. S. Progress in Crystal Physics. Vol. 1, 1958.

11. McCormick, John M. and Salvadori, Mario G. Numerical Methods in Fortran. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1964.

APPENDIX A

TABLE A-1

$h^n f(n)$	Multiplier	Differentiation Formula Coefficients				
	1	(-1)	1			
	1/2	(-3)	4	-1		
	1/6	(-11)	18	-9	2	
	1/12	(-25)	48	-36	16	-3
hf'	1/2	-1	(0)	1		
	1/6	-2	(-3)	6	-1	
	1/12	-3	(-10)	18	-6	1
	1/12	1	-8	(0)	8	-1
	1	(1)	-2	1		
	1	(2)	-5	4	-1	
	1/12	(35)	-104	114	-56	11
$h^2 f''$	1	1	(-2)	1		
	1	1	(-2)	1	0	
	1/12	11	(-20)	6	4	-1
	1/12	-1	16	(-30)	16	-1
	1	(-1)	3	-3	1	
	1/2	(-5)	18	-24	14	-3
$h^3 f'''$	1	-1	(3)	-3	1	
	1/2	-3	(10)	-12	6	-1
	1/2	-1	2	(0)	-2	1
	1	(1)	-4	6	-4	1
$h^4 f^{IV}$	1	1	(-4)	6	-4	1
	1	1	-4	(6)	-4	1

The number within the parentheses is the coefficient of the (i,j)th term of the finite difference expression. Those to its right are the coefficients of the finite difference terms in the forward direction, and the others are the coefficients of the finite difference terms in the backward direction.

Computer Program Listing

```
C
C        THE PROGRAM IS DESIGNED TO COMPUTE THERMAL STRESSES
C        AND DEFLECTIONS IN AN ANISOTROPIC PLATE. THE PLATE
C        DIMENSIONS ARE REPRESENTED BY NX, NY, AND DELH, DELK.
C        ADJUSTING THESE PARAMETERS DIFFERENT SIZES OF PLATE
C        CAN BE ANALYZED FOR THERMAL STRESSES. IF NEED BE, THE
C        DIMENSIONS OF F, TEMF, ETC.. IN THE DIMENSION STATEMENT
C        MAY ALSO BE CHANGED.
C
      DIMENSION F(30,30),TEMF(30,30),ANX(30,30),ANY(30,30)
      DIMENSION ANXY(30,30),W(30,30),TEMPW(30,30)
      DIMENSION TELX(30,30),TELY(30,30),TELXY(30,30)
      DIMENSION T0(30,30),T1(30,30),CURL1X(30,30),CURL1Y(30,30)
      DIMENSION A(6,6),AT(6,6),C(6,6),B(6,6)
      DIMENSION ALFA(6),K(6),G(6)
      INTEGER G
C
C       READ IN COMPLIANCES, DIMENSIONS OF PLATE, INCREMENTS
C       ALONG X AND Y DIRECTIONS, AND LINEAR COEFFICIENTS OF
C       THERMAL EXPANSION .
      READ(5,15)  NX,NY,DELH,DELK,H
      DO 700 I=1,6
  700 READ(5,710) (A(I,J),J=1,6)
      DO 6  I=1,6
    6 WRITE (6,8) (A(I,J),J=1,6)
    8 FORMAT(10X,6F15.4)
      READ(5,10) AA1,AA2,AA3,AA6
      N=6
C
      A1=AA1*1.E-6
      A2=AA2*1.E-6
      A3=AA3*1.E-6
```

```
      A6=AA6*1.E-6
C
C        COMPUTE THE ELASTIC CONSTANTS C(I.J).
C
      DO 600 I=1.6
      DO 600 J=1.6
 600  AT(I.J)=A(I.J)
      CALL MINV(AT.N.D.K.G)
C
C        MINV IS A SUBROUTINE AVAILABLE IN THE SYSLIB.
C        THIS SUBROUTINE GIVES THE VALUES OF DETERMINENT
C        AND THE COFACTORS OF THE ELEMENTS A(I.J)S' OF THE MATRIX A.
      DO 610 I=1.6
      DO 610 J=1.6
      DO 620 L=1.6
      DO 620 M=1.6
  620 AT(L.M)=A(L.M)
      DO 630 M=1.6
      IF(M.EQ.J) GO TO 640
      AT(I.M)=0.
      GO TO 630
 640  AT(I.M)=1.
 630  CONTINUE
      CALL MINV(AT.N.D1.K.G)
      L=I+J
      DO 650 M=1.L
 650  D1=D1*(-1)
      C(I.J)=D1/D
 610  CONTINUE
C
C       DETERMINE THE CONSTANTS B(I.J) AND MODIFIED THERMAL
C       EXPANSION COEFFICIENTS ALFAS'.
```

```
C
      DO 720 I=1.6
      IF( I.GT.2.AND.I.LT.6) GO TO 720
      DO 730 J=1.6
      IF( J.GT.2.AND.J.LT.6) GO TO 730
      B(I.J)=(C(I.J)-C(I.3)*(C(J.3)/C(3.3)))*1.E+8
  730 CONTINUE
      ALFA(I)=B(1.I)*A1+B(2.I)*A2+B(6.I)*A6
  720 CONTINUE
      H1=3./(2*H**3.)
      ALFA1=ALFA(1)
      ALFA2=ALFA(2)
      ALFA6=ALFA(6)
C
      CALL TEMDIS(DELH.DELK.NX.NY.T0.T1.CURL1X.CURL1Y.H)
C
      CALL FUNCF(A.A1.A2.A3.A6.NX.NY.DELH.DELK.F.TEMF)
C
      CALL SIGMA0 (F.NX.NY.H.DELH.DELK.ANX.ANY.ANXY)
C
      CALL DEFLXN (B.ANX.ANY.ANXY.NX.NY.DELH.DELK.W.H1.
     . ALFA1.ALFA2.ALFA3.ALFA6.TEMPW.T1.CURL1X.CURL1Y.ALFA)
C
      CALL SIGMA1(W.B.DELH.DELK.H.NX.NY.TELX.TELY.TELXY.ALFA.T1)
C
C       COMPUTE TOTAL STRESS DISTRIBUTION ON THE UPPER SURFACE
      DO 900 I=1.NX
      DO 900 J=1.NY
      ANX(I.J)=ANX(I.J)+TELX(I.J)
      ANY(I.J)=ANY(I.J)+TELY(I.J)
  900 ANXY(I.J)=ANXY(I.J)+TELXY(I.J)
C
```

```
C          PRINT TOTAL  STRESS DISTRIBUTION ON THE UPPER SURFACE
       WRITE (6,155)   (J,J=1,6)
       DO 160 I=1,NX
  160  WRITE (6,210)   I,(ANX(I,J),J=1,NY)
       WRITE (6,165)   (J,J=1,6)
       DO 172 I=1,NX
  172  WRITE (6,210)   I,(ANY(I,J),J=1,NY)
       WRITE (6,175)   (J,J=1,6)
       DO 180 I=1,NX
  180  WRITE (6,210)   I,(ANXY(I,J),J=1,NY)
   10  FORMAT(4F10.4)
   15  FORMAT(2I6,3F8.3)
  155  FORMAT(1H1////////////1X,T30,'STRESS SIGMAX ON THE UPPER SURFACE (P
      .SI)'///17X,6(I2,12X)/)
  165  FORMAT(1H1////////////1X,T30,'STRESS SIGMAY ON THE UPPER SURFACE (P
      .SI)'///17X,6(I2,12X)/)
  175  FORMAT(1H1////////////1X,T30,'STRESS SIGMAXY ON THE UPPER SURFACE (
      .PSI)'///17X,6(I2,12X)/)
  210   FORMAT(1H,I12,(T15,6E14.5))
   710 FORMAT(6F10.4)
       STOP
       END


       SUBROUTINE FUNCF(A,A1,A2,A3,A6,NX,NY,DELH,DELK,F,TEMF)
       DIMENSION F(30,30),TEMF(30,30),A(6,6),ANXY(30,30)
       DIMENSION  T0(30,30),T1(30,30),RHS(30,30)
       X(Z)=(Z-1.)*DELH
       Y(Z)=(Z-1.)*DELK
C
C          RIGHT HAND SIDE FUNCTION RHSF(X,Y).
```

```
      RHSF(X,Y)= ((A2*(12.*U1*X*X+6.*(U2*X*Y+U3*X)+2.*(U4*Y*Y
     .   +U5*Y+U6))+A1*(2.*U4*X*X+2.*U10+2.*U11*X+6.*U12*Y
     .   +6.*U13*X*Y+12.*U14*Y*Y)-2.*A6*(3.*U2*X*X+4.*U4*X*Y
     .   +2.*U5*X+U8+2.*U11*Y+3.*U13*Y*Y))*1.E+8)
      EPSLNF=1.E-2
      READ(5,10) U1,U2,U3,U4,U5,U6,U7,U8,U9,U10,U11,U12,U13,
     C           U14
      NX2=NX-2
      NY2=NY-2
C
C        COMPUTE THE CONSTANTS P,Q,R,S,T,AND BMS'.
C
      P=A(2,2)/(DELH**4)
      Q=A(2,6)/(12.*DELH**3*DELK)
      R= (2.*A(1,2)+A(6,6))/(144.*DELH**2*DELK**2)
      S=A(1,6)/(12.*DELH*DELK**3)
      T=A(1,1)/(DELK**4)
C
      BM1=Q+R+S
      BM2=-8.*Q-16.*R-2.*S
      BM3=P+30.*R
      BM4=8.*Q-16.*R+2.*S
      BM5=-Q+R-S
      BM6=-2.*Q-16.*R-8.*S
      BM7=16.*Q+256.*R+16.*S
      BM8=-4.*P-480.*R
      BM9=-16.*Q+256.*R-16.*S
      BM10=2.*Q-16.*R+8.*S
      BM11=30.*R+T
      BM12=-480.*R-4.*T
      BM13=6.*P+900.*R+6.*T
      BM14=BM12
```

```
      BM15=BM11
      BM16=BM10
      BM17=BM9
      BM18=BM8
      BM19=BM7
      BM20=BM6
      BM21=BM5
      BM22=BM4
      BM23=BM3
      BM24=BM2
      BM25=BM1
C
C        INITIALIZE F, AND TEMF AND ESTABLISH THE BOUNDARY
C        CONDITIONS.
C
      DO 1500  I=1,NX
      DO 1500 J=1,NY
      F(I,J)=0.
 1500  TEMF(I,J)=0.
      M=1
 100  DO 90 I=3,NX2
      DO 90 J=3,NY2
      U=I
      V=J
      TEMF(I,J)=-(BM1*F(I-2,J-2)+BM2*F(I-2,J-1)+BM3*F(I-2,J)+
     .BM4*F(I-2,J+1)+BM5*F(I-2,J+2)+BM6*F(I-1,J-2)+BM7*F(I-1,J-1)
     .+BM8*F(I-1,J)+BM9*F(I-1,J+1)+BM10*F(I-1,J+2)+BM11*F(I,J-2)
     .+BM12*F(I,J-1)+BM14*F(I,J+1)+BM15*F(I,J+2)+BM16*F(I+1,J-2)
     .+BM17*F(I+1,J-1)+BM18*F(I+1,J)+BM19*F(I+1,J+1)+BM20*
     .F(I+1,J+2)+BM21*F(I+2,J-2)+BM22*F(I+2,J-1)+BM23*F(I+2,J)+
     .BM24*F(I+2,J+1)+BM25*F(I+2,J+2)+RHSF(X(U),Y(V)))/BM13
  90  CONTINUE
```

```
C
C          CHECK FOR CONVERGENCE OF F.
      DO 120 I=1.NX
      DO 120 J=1.NY
      IF (ABS(TEMF(I.J)-F(I.J)).GT.EPSLNF) GO TO 140
  120 CONTINUE
      GO TO 150
  140 M=M+1
      DO 1010 I=1.NX
      DO 1010 J=1.NY
 1010 F(I.J)=TEMF(I.J)
      IF (M.LE.40)  GO TO 100
   10 FORMAT(8F10.4)
  150 RETURN
      END


      SUBROUTINE SIGMAO (F.NX.NY.H.DELH.DELK.ANX.ANY.ANXY)
      DIMENSION F(30.30).ANX(30.30).ANY(30.30).ANXY(30.30)
C
C          ANX. ANY. ANXY RESPECTIVELY REPRESENT STRESSES SIGMAX-0.
C          SIGMAY-0. AND SIGMAXY-0.
      NXA=NX-4
      DO 120 I=1.NXA
      DO 120 J=1.4
      ANX(I.J)=(F(I.J)-2.*F(I.J+1)+F(I.J+2))/DELK**2.
      ANY(I.J)=(F(I.J)-2.*F(I+1.J)+F(I+2.J))/DELH**2.
  120 ANXY(I.J)=(F(I.J)-F(I.J+1)-F(I+1.J)+F(I+1.J+1))/(DELH*DELK)
      DO 130 I=1.4
      DO 130 J=5.NY
      ANX(I.J)=(F(I.J)-2.*F(I.J-1)+F(I.J-2))/DELK**2.
```

```
      ANY(I,J)=(F(I,J)-2.*F(I+1,J)+F(I+2,J))/DELH**2.
  130 ANXY(I,J)=-(F(I,J)-F(I,J-1)-F(I+1,J)+F(I+1,J-1))/(DELH*DELK)
      NYA=NY-3
      DO 140 I=5,NX
      DO 140 J=NYA ,NY
      ANX(I,J)=(F(I,J)-2.*F(I,J-1)+F(I,J-2))/DELK**2.
      ANY(I,J)=(F(I,J)-2.*F(I-1,J)+F(I-2,J))/DELH**2.
  140 ANXY(I,J)=(F(I,J)-F(I,J-1)-F(I-1,J)+F(I-1,J-1))/(DELH*DELK)
      NXA=NX-3
      NYA=NY-4
      DO 145 I=NXA ,NX
      DO 145 J=1,NYA
      ANX(I,J)=(F(I,J)-2.*F(I,J+1)+F(I,J+2))/DELK**2.
      ANY(I,J)=(F(I,J)-2.*F(I-1,J)+F(I-2,J))/DELH**2.
  145 ANXY(I,J)=-(F(I,J)-F(I,J+1)-F(I-1,J)+F(I-1,J+1))/(DELH*DELK)
      NXA=NX-4
      DO 150 I= 5,NXA
      DO 150 J= 5,NYA
      ANX(I,J)=2*H*(-F(I,J-2)+16.*F(I,J-1)-30.*F(I,J)+16.*
     .F(I,J+1)-F(I,J+2))/(12.*DELK**2.)
      ANY(I,J)=2*H*(-F(I-2,J)+16.*F(I-1,J)-30.*F(I,J)+16.*
     .F(I+1,J)-F(I+2,J))/(12.*DELH**2.)
  150 ANXY(I,J)=2*H*(F(I-1,J-1)-F(I-1,J+1)-F(I+1,J-1)
     .+F(I+1,J+1))/(4.*DELH*DELK)
C
C        PRINT THE STRESSES SIGMAX-0. SIGMAY-0. SIGMAXY-0.
      WRITE (6,155)  (J,J=1,6)
      DO 160 I=1,NX
  160 WRITE (6,210)  I,(ANX(I,J),J=1,NY)
      WRITE (6,165)  (J,J=1,6)
      DO 172 I=1,NX
  172 WRITE (6,210)  I,(ANY(I,J),J=1,NY)
```

```
      WRITE (6,175)  (J,J=1,6)
      DO 180 I=1,NX
 180  WRITE (6,210)  I,(ANXY(I,J),J=1,NY)
C
C        ANX, ANY, ANXY RESPECTIVELY REPRESENT FORCES NX, NY, NXY,
C        FROM HERE ON.
      DO 170 I=1,NX
      DO 170 J=1,NY
      ANX(I,J)=ANX(I,J)*2.*H
      ANY(I,J)=ANY(I,J)*2.*H
 170  ANXY(I,J)=ANXY(I,J)*2.*H
 155  FORMAT(1H1///////////1X,T40,'STRESS SIGMAX-0 (PSI)'///
     .   17X,6(I2,12X)/)
 165  FORMAT(1H1///////////1X,T40,'STRESS SIGMAY-0 (PSI)'///
     .   17X,6(I2,12X)/)
 175  FORMAT(1H1///////////1X,T40,'STRESS SIGMAXY-0 (PSI)'///
     .   17X,6(I2,12X)/)
 210   FORMAT(1H,I12,(T15,6E14.5))
      RETURN
      END


      SUBROUTINE TEMDIS (DELH,DELK,NX,NY,T0,T1,CURL1X,CURL1Y,H)
      DIMENSION T0(30,30),T1(30,30),CURL1X(30,30),CURL1Y(30,30)
C
C        THIS MODULE GIVES T0 AND T1 DISTRIBUTION FOR WHICH
C        FOURTH ORDER POLYNOMIALS ARE ASSUMED.
      READ(5,10) U1,U2,U3,U4,U5,U6,U7,U8,U9,U10,U11,U12,U13,
     .           U14,U15
      READ(5,10) V1,V2,V3,V4,V5,V6,V7,V8,V9,V10,V11,V12,V13,
     .           V14,V15
```

```
      DO 20 I=1,NX
      DO 20 J=1,NY
      U=I
      V=J
      X=(U-1.)*DELH
      Y=(V-1.)*DELK
      T0(I,J)=U1*X**4+U2*X**3*Y+U3*X**3+U4*X*X*Y*Y+U5*X*X*Y+U6*X*X
     .   +U7*X+U8*X*Y+U9*Y+U10*Y*Y+U11*X*Y*Y+U12*Y**3+U13*X*Y**3
     .  +U14*Y**4+CONST1
      T1(I,J)=(V1*X**4+V2*X**3*Y+V3*X**3+V4*X*X*Y*Y+V5*X*X*Y+V6*X*X
     .   +V7*X+V8*X*Y+V9*Y+V10*Y*Y+V11*X*Y*Y+V12*Y**3+V13*X*Y**3
     .  +V14*Y**4+CONST2)*H
C
C        CURL1X, AND CURL1Y ARE FIRST DERIVATIVES OF T1 WITH
C        RESPECT TO X, AND Y, RESPECTIVELY.
      CURL1X(I,J)=4.*V1*X**3+3.*V2*X*X*Y+3.*V3*X*X+2.*V4*X*Y*Y
     .   +2.*V5*X*Y+2.*V6*X+V7+V8*Y+V11*Y*Y+V13*Y**3
      CURL1Y(I,J)=V2*X**3+2.*V4*X*X*Y+V5*X*X+V8*X+V9+2.*V10*Y+2.*V11
     .   *X*Y+3.*V12*Y*Y+3.*V13*X*Y*Y+4.*V14*Y**3
   20 CONTINUE
      WRITE(6,15)  (J,J=1,6)
      DO 30 I=1,NX
   30 WRITE(6,60)  I,(T0(I,J),J=1,NY)
      WRITE (6,23)  (J,J=1,6)
      DO 40 I=1,NX
   40 WRITE(6,60)  I,(T1(I,J),J=1,NY)
      DO 70 I=1,NX
      DO 70 J=1,NY
   70 T1(I,J)=T1(I,J)/H
   10 FORMAT(8F10.4)
   15 FORMAT(1H1//////////1X,T35,'T0 TEMPERATURE DISTRIBUTION (DEGREES C
     .)'///17X,6(I2,11X)/)
```

```
23   FORMAT(1H1///////////1X,T15,'TEMPERATURE DISTRIBUTION DUE TO ZT1 ON
    . THE SURFACE (DEGREES C)'///17X,6(I2,11X)/)
60   FORMAT(1H,I12,(T15,6E13.4))
     RETURN
     END


     SUBROUTINE DEFLXN (B,ANX,ANY,ANXY,NX,NY,DELH,DELK,W,H1,
    . ALFA1,ALFA2,ALFA3,ALFA6,TEMPW,TT1,CURL1X,CURL1Y,ALFA)
     DIMENSION B(6,6),ANX(30,30),ANY(30,30),ANXY(30,30)
     DIMENSION TT1(30,30),CURL1X(30,30),CURL1Y(30,30)
     DIMENSION W(30,30),TEMPW(30,30),ALFA(6)
     X(Z)= (Z-1.)*DELH
     Y(Z)= (Z-1.)*DELK
C
C       RIGHT HAND SIDE FUNCTION RHSW(X,Y).
     RHSW(X,Y)= ALFA1*(12.*V1*X*X+6.*(V2*X*Y+V3*X)+2.*(V4*Y*Y
    .    +V5*Y+V6))+ALFA2*(2.*(V4*X*X+V10+V11*X)+6.*(V12*Y
    .    +V13*X*Y)+12.*V14*Y*Y)+2.*ALFA6*(3.*V2*X*X+4.*V4*X*Y
    .    +2.*V5*X+V8+2.*V11*Y+3.*V13*Y*Y)
     READ(5,10) V1,V2,V3,V4,V5,V6,V7,V8,V9,V10,V11,V12,V13,
    .          V14
     EPSLNW=1.E-5
     NX4=NX-4
     NX3=NX-3
     NX2=NX-2
     NX1=NX-1
     NY4=NY-4
     NY3=NY-3
     NY2=NY-2
     NY1=NY-1
```

```
      P1=B(1,1)/DELH**4
      Q1=-3.*B(1,6)/(24.*DELH**3*DELK)
      R1=2.*(B(1,2)+B(6,6))/(144.*DELH**2*DELK**2)
      S1=-3.*B(2,6)/(24.*DELH*DELK**3)
      T1=B(2,2)/DELK**4
C
C     THE COEFFICIENTS OF THE FINITE DIFFERENCE EQUATIONS FOR
C     COMPUTING THE DEFLECTION.
      DM1=Q1+R1+S1
      DM2=-8.*Q1-16.*R1-2.*S1
      DM3=P1+30.*R1
      DM4=8.*Q1-16.*R1+2.*S1
      DM5=-Q1+R1-S1
      DM6=-2.*Q1-16.*R1-8.*S1
      DM7=16.*Q1+256.*R1+16.*S1
      DM8=-4.*P1-480.*R1
      DM9=-16.*Q1+256.*R1-16.*S1
      DM10=2.*Q1-16.*R1+8.*S1
      DM11=30*R1+T1
      DM12=-480.*R1-4.*T1
      DM13=6.*P1+900.*R1+6.*T1
      DM14=DM12
      DM15=DM11
      DM16=DM10
      DM17=DM9
      DM18=DM8
      DM19=DM7
      DM20=DM6
      DM21=DM5
      DM22=DM4
      DM23=DM3
      DM24=DM2
```

```
      DM25=DM1
C
C        COEFFICIENTS OF THE FINITE DIFFERENCE EQUATION
C        REPRESENTING BOUNDARY CONDITION.
C
       B1= B(1,1)/DELH**3
      B2= -(B(1,6)/(DELH**2*DELK))
      B3= (B(1,2)-2.*B(6,6))/(DELH*DELK**2)
      B4= -2.*B(2,6)/DELK**3
      B11=2.*B(1,6)/DELH**3
      B22=(2.*B(1,6)-B(1,2))/(DELH**2*DELK)
      B33=B(2,6)/(DELH*DELK**2)
      B44=B(2,2)/DELK**3
      BT1=B(1,1)/DELH**2
      BT2=B(1,2)/DELK**2
      BT3=B(2,6)/(DELH*DELK)
      BT11=B(1,2)/DELH**2
      BT22=B(2,2)/DELK**2
      BT33=B(2,6)/(DELH*DELK)
C
C         COMPUTE VALUES OF W ON BOUNDARIES.
      DO 20 I=1,NX
      DO 20 J=1,NY
      W(I,J)=0.
   20 TEMPW(I,J)=0.
      M=1
  400 DO 100 J=1,NY3
      I=1
      TEMPW(I,J)= ((2.*BT2+BT3)*W(I,J+1)-BT2*W(I,J+2)+(2.*BT1+BT3)*
     .   W(I+1,J)-BT3*W(I+1,J+1)-BT1*W(I+2,J)-ALFA(1)*TT1(I,J))/
     .   (BT1+BT2+BT3)
      TEMPW(I+1,J)= ((B1+B2+B3+B4)*W(I,J)-(B2+2.*B3+3.*B4)*W(I,J+1)
```

```
     .   +(B3+3.*B4)*W(I,J+2)-B4*W(I,J+3)+2.*(B2+B3)*W(I+1,J+1)
     .   -B3*W(I+1,J+2)+(3.*B1+B2)*W(I+2,J)-B2*W(I+2,J+1)
     .   -B1*W(I+3,J)-(ALFA(1)*CURL1X(I,J)-2.*ALFA(6)*
     .   CURL1Y(I,J)))/(3.*B1+2.*B2+B3)
      I=NX
      TEMPW(I,J)= ((2.*BT2-BT3)*W(I,J+1)-BT2*W(I,J+2)+(2.*BT1-BT3)*
     .   W(I-1,J)+BT3*W(I-1,J+1)-BT1*W(I-2,J)-ALFA(1)*TT1(I,J))/
     .   (BT1+BT2-BT3)
      TEMPW(I-1,J)= ((B1+B2+B3+B4)*W(I,J)-(B2+2.*B3+3.*B4)*W(I,J+1)
     .   +(B3+3.*B4)*W(I,J+2)-B4*W(I,J+3)+2.*(B2+B3)*W(I-1,J+1)
     .   -B3*W(I-1,J+2)+(3.*B1+B2)*W(I-2,J)-B2*W(I-2,J+1)
     .   -B1*W(I-3,J)+(ALFA(1)*CURL1X(I,J)-2.*ALFA(6)*
     .   CURL1Y(I,J)))/(3.*B1+2.*B2+B3)
100   CONTINUE
      DO 120 J=NY2,NY
      I=1
      TEMPW(I,J)= ((2.*BT2-BT3)*W(I,J-1)-BT2*W(I,J-2)+(2.*BT1-BT3)*
     .   W(I+1,J)+BT3*W(I+1,J-1)-BT1*W(I+2,J)-ALFA(1)*TT1(I,J))/
     .   (BT1+BT2-BT3)
      TEMPW(I+1,J)= ((B1+B2+B3+B4)*W(I,J)-(B2+2.*B3+3.*B4)*W(I,J-1)
     .   +(B3+3.*B4)*W(I,J-2)-B4*W(I,J-3)+2.*(B2+B3)*W(I+1,J-1)
     .   -B3*W(I+1,J-2)+(3.*B1+B2)*W(I+2,J)-B2*W(I+2,J-1)
     .   -B1*W(I+3,J)+(ALFA(1)*CURL1X(I,J)-2.*ALFA(6)*
     .   CURL1Y(I,J)))/(3.*B1+2.*B2+B3)
      I=NX
      TEMPW(I,J)= ((2.*BT2+BT3)*W(I,J-1)-BT2*W(I,J-2)+(2.*BT1+BT3)*
     .   W(I-1,J)-BT3*W(I-1,J-1)-BT1*W(I-2,J)-ALFA(1)*TT1(I,J))/
     .   (BT1+BT2+BT3)
      TEMPW(I-1,J)= ((B1+B2+B3+B4)*W(I,J)-(B2+2.*B3+3.*B4)*W(I,J-1)
     .   +(B3+3.*B4)*W(I,J-2)-B4*W(I,J-3)+2.*(B2+B3)*W(I-1,J-1)
     .   -B3*W(I-1,J-2)+(3.*B1+B2)*W(I-2,J)-B2*W(I-2,J-1)
     .   -B1*W(I-3,J)-(ALFA(1)*CURL1X(I,J)-2.*ALFA(6)*
```

```
     .    CURL1Y(I,J)))/(3.*B1+2.*B2+B3)
120   CONTINUE
      DO 150 I=2,NX3
      J=1
      TEMPW(I,J)= ((2.*BT22+BT33)*W(I,J+1)-BT22*W(I,J+2)+(2.*BT11+
     .    BT33)*W(I+1,J)-BT33*W(I+1,J+1)-BT11*W(I+2,J)-ALFA(2)*
     .    TT1(I,J))/(BT11+BT22+BT33)
      TEMPW(I,J+1)= ((B11+B22+B33+B44)*W(I,J)+(B33+3.*B44)*W(I,J+2)
     .    -B44*W(I,J+3)-(3.*B11+2.*B22+B33)*W(I+1,J)+
     .    2.*(B22+B33)*W(I+1,J+1)-B33*W(I+1,J+2)+(3.*B11+B22)*
     .    W(I+2,J)-B22*W(I+2,J+1)-B11*W(I+3,J)-(2.*ALFA(6)*
     .    CURL1X(I,J)-ALFA(2)*CURL1Y(I,J)))/(B22+2.*B33+3.*B44)
      J=NY
      TEMPW(I,J)= ((2.*BT22-BT33)*W(I,J-1)-BT22*W(I,J-2)+(2.*BT11-
     .    BT33)*W(I+1,J)+BT33*W(I+1,J-1)-BT11*W(I+2,J)-ALFA(2)*
     .    TT1(I,J))/(BT11+BT22-BT33)
      TEMPW(I,J-1)= ((B11+B22+B33+B44)*W(I,J)+(B33+3.*B44)*W(I,J-2)
     .    -B44*W(I,J-3)-(3.*B11+2.*B22+B33)*W(I+1,J)+
     .    2.*(B22+B33)*W(I+1,J-1)-B33*W(I+1,J-2)+(3.*B11+B22)*
     .    W(I+2,J)-B22*W(I+2,J-1)-B11*W(I+3,J)+(2.*ALFA(6)*
     .    CURL1X(I,J)-ALFA(2)*CURL1Y(I,J)))/(B22+2.*B33+3.*B44)
150   CONTINUE
      DO 160 I=NX2,NX1
      J=1
      TEMPW(I,J)= ((2.*BT22-BT33)*W(I,J+1)-BT22*W(I,J+2)+(2.*BT11-
     .    BT33)*W(I-1,J)+BT33*W(I-1,J+1)-BT11*W(I-2,J)-ALFA(2)*
     .    TT1(I,J))/(BT11+BT22-BT33)
      TEMPW(I,J+1)= ((B11+B22+B33+B44)*W(I,J)+(B33+3.*B44)*W(I,J+2)
     .    -B44*W(I,J+3)-(3.*B11+2.*B22+B33)*W(I-1,J)+
     .    2.*(B22+B33)*W(I-1,J+1)-B33*W(I-1,J+2)+(3.*B11+B22)*
     .    W(I-2,J)-B22*W(I-2,J+1)-B11*W(I-3,J)+(2.*ALFA(6)*
     .    CURL1X(I,J)-ALFA(2)*CURL1Y(I,J)))/(B22+2.*B33+3.*B44)
```

```
      J=NY
      TEMPW(I,J)= ((2.*BT22+BT33)*W(I,J-1)-BT22*W(I,J-2)+(2.*BT11+
     .   BT33)*W(I-1,J)-BT33*W(I-1,J-1)-BT11*W(I-2,J)-ALFA(2)*
     .   TT1(I,J))/(BT11+BT22+BT33)
      TEMPW(I,J-1)= ((B11+B22+B33+B44)*W(I,J)+(B33+3.*B44)*W(I,J-2)
     .   -B44*W(I,J-3)-(3.*B11+2.*B22+B33)*W(I-1,J)+
     .   2.*(B22+B33)*W(I-1,J-1)-B33*W(I-1,J-2)+(3.*B11+B22)*
     .   W(I-2,J)-B22*W(I-2,J-1)-B11*W(I-3,J)-(2.*ALFA(6)*
     .   CURL1X(I,J)-ALFA(2)*CURL1Y(I,J)))/(B22+2.*B33+3.*B44)
 160  CONTINUE
C
C        COMPUTE DEFLECTION AT THE INNER POINTS.
C
      DO 350 J=3,NY2
      DO 350 I=3,NX2
      U=I
      V=J
 350  TEMPW(I,J)=-(DM1*W(I-2,J-2)+DM2*W(I-2,J-1)+DM3*W(I-2,J)
     .+DM4*W(I-2,J+1)+DM5*W(I-2,J+2)+DM6*W(I-1,J-2)+DM7*
     .W(I-1,J-1)+(DM8-H1*ANX(I,J)/(DELH**2.))*W(I-1,J)+DM9*
     .W(I-1,J+1)+DM10*W(I-1,J+2)+DM11*W(I,J-2)+(DM12-H1*
     .ANY(I,J)/(DELK**2.))*W(I,J-1)+(DM14-H1*ANY(I,J)/
     .(DELK**2.))*W(I,J+1)+DM15*W(I,J+2)+DM16*W(I+1,J-2)+DM17
     .*W(I+1,J-1)+(DM18-H1*ANX(I,J)/(DELH**2.))*W(I+1,J)+
     .DM19*W(I+1,J+1)+DM20*W(I+1,J+2)+DM21*W(I+2,J-2)+DM22*
     .W(I+2,J-1)+DM23*W(I+2,J)+DM24*W(I+2,J+1)+DM25*W(I+2,J+2)
     .-H1*ANXY(I,J)*(W(I-1,J-1)-W(I-1,J+1)-W(I+1,J-1)+
     .W(I+1,J+1))/(2.*DELH*DELK)+RHSW(X(U),Y(V)))/(DM13+H1*
     .2.*(ANX(I,J)/(DELH**2.)+ANY(I,J)/(DELK**2.)))
C
C        CHECK FOR CONVERGENCE OF W.
      DO 370 I=1,NX
```

```
      DO 370 J=1,NY
      IF(ABS(TEMPW(I,J)-W(I,J)).GT.EPSLNW) GO TO 375
  370 CONTINUE
  375 M=M+1
C
C        IF NO CONVERGENCE REPLACE OLD VALUES OF W WITH NEW
C        VALUES OF W, THAT IS TEMPW.
      DO 390 I=1,NX
      DO 390 J=1,NY
  390 W(I,J)=TEMPW(I,J)
      IF (M.GT.40)  GO TO 380
      GO TO 400
  380 WRITE (6,440)  (J,J=1,6)
      DO 395 I=1,NX
  395 WRITE (6,500)  I,(W(I,J),J=1,NY)
   10 FORMAT(8F10.4)
  440 FORMAT (1H1///////////1H,T35,'DEFLECTION (INCH)'///
     .   17X,6(I2,11X))
  500 FORMAT (1H,I12,(T15,6E13.4))
      RETURN
      END


      SUBROUTINE SIGMA1(W,B,DELH,DELK,H,NX,NY,TELX,TELY,TELXY,
     .   ALFA,T1)
      DIMENSION W(30,30),TELX(30,30),TELY(30,30),TELXY(30,30)
      DIMENSION  T1(30,30),B(6,6),ALFA(6)
C
C        TELX, TELY, AND TELXY COSTITUTE THE STRESS TENSOR TAU-1.
      NX4=NX-4
      NX3=NX-3
```

```
      NX2=NX-2
      NX1=NX-1
      NY4=NY-4
      NY3=NY-3
      NY2=NY-2
      NY1=NY-1
C        COEFFICIENTS OF THE FINITE DIFFERENCE EQUATIONS FOR
C        STRESSES TAU-1.
      P2=B(1,1)/(12.*DELH**2.)
      Q2=B(1,2)/(12.*DELK**2.)
      R2=B(1,6)/(4.*DELH*DELK)
      P3=B(1,2)/(12.*DELH**2.)
      Q3=B(2,2)/(12.*DELK**2.)
      R3=B(2,6)/(4.*DELH*DELK)
      P4=B(1,6)/(12.*DELH**2.)
      Q4=B(2,6)/(12.*DELK**2.)
      R4=B(6,6)/(4.*DELH*DELK)
      DO 800 I=1,NX4
C
C        STRESS COMPUTATION.
      DO 800 J=1,4
      TELX(I,J)=-H*((35.*(P2+Q2)+9.*R2)*W(I,J)-(104.*Q2+12.*R2)
     . *W(I,J+1)+(114.*Q2+3.*R2)*W(I,J+2)-56.*Q2*W(I,J+3)+
     . 11.*Q2*W(I,J+4)-(104.*P2+12.*R2)*W(I+1,J)+16.*R2*W(I+1,J+1)
     . -4.*R2*W(I+1,J+2)+(114.*P2+3.*R2)*W(I+2,J)-4.*R2*
     . W(I+2,J+1)+R2*W(I+2,J+2)-56.*P2*W(I+3,J)+11.*P2*W(I+4,J))
      TELY(I,J)=-H*((35.*(P3+Q3)+9.*R3)*W(I,J)-(104.*Q3+12.*R3)
     . *W(I,J+1)+(114.*Q3+3.*R3)*W(I,J+2)-56.*Q3*W(I,J+3)+
     . 11.*Q3*W(I,J+4)-(104.*P3+12.*R3)*W(I+1,J)+16.*R3*W(I+1,J+1)
     . -4.*R3*W(I+1,J+2)+(114.*P3+3.*R3)*W(I+2,J)-4.*R3*
     . W(I+2,J+1)+R3*W(I+2,J+2)-56.*P3*W(I+3,J)+11.*P3*W(I+4,J))
      TELXY(I,J)=-H*((35.*(P4+Q4)+9.*R4)*W(I,J)-(104.*Q4+12.*R4)
```

```
     . *W(I,J+1)+(114.*Q4+3.*R4)*W(I,J+2)-56.*Q4*W(I,J+3)+
     . 11.*Q4*W(I,J+4)-(104.*P4+12.*R4)*W(I+1,J)+16.*R4*W(I+1,J+1)
     . -4.*R4*W(I+1,J+2)+(114.*P4+3.*R4)*W(I+2,J)-4.*R4*
     . W(I+2,J+1)+R4*W(I+2,J+2)-56.*P4*W(I+3,J)+11.*P4*W(I+4,J))
800   CONTINUE
      DO 810 I=1,4
      DO 810 J=5,NY
      TELX(I,J)=-H*((35.*(P2+Q2)-9.*R2)*W(I,J)-(104.*Q2-12.*R2)
     . *W(I,J-1)+(114.*Q2-3.*R2)*W(I,J-2)-56.*Q2*W(I,J-3)+
     . 11.*Q2*W(I,J-4)-(104.*P2-12.*R2)*W(I+1,J)-16.*R2*W(I+1,J-1)
     . +4.*R2*W(I+1,J-2)+(114.*P2-3.*R2)*W(I+2,J)+4.*R2*
     . W(I+2,J-1)-R2*W(I+2,J-2)-56.*P2*W(I+3,J)+11.*P2*W(I+4,J))
      TELY(I,J)=-H*((35.*(P3+Q3)-9.*R3)*W(I,J)-(104.*Q3-12.*R3)
     . *W(I,J-1)+(114.*Q3-3.*R3)*W(I,J-2)-56.*Q3*W(I,J-3)+
     . 11.*Q3*W(I,J-4)-(104.*P3-12.*R3)*W(I+1,J)-16.*R3*W(I+1,J-1)
     . +4.*R3*W(I+1,J-2)+(114.*P3-3.*R3)*W(I+2,J)+4.*R3*
     . W(I+2,J-1)-R3*W(I+2,J-2)-56.*P3*W(I+3,J)+11.*P3*W(I+4,J))
      TELXY(I,J)=-H*((35.*(P4+Q4)-9.*R4)*W(I,J)-(104.*Q4-12.*R4)
     . *W(I,J-1)+(114.*Q4-3.*R4)*W(I,J-2)-56.*Q4*W(I,J-3)+
     . 11.*Q4*W(I,J-4)-(104.*P4-12.*R4)*W(I+1,J)-16.*R4*W(I+1,J-1)
     . +4.*R4*W(I+1,J-2)+(114.*P4-3.*R4)*W(I+2,J)+4.*R4*
     . W(I+2,J-1)-R4*W(I+2,J-2)-56.*P4*W(I+3,J)+11.*P4*W(I+4,J))
810   CONTINUE
      DO 820 I=5,NX
      DO 820 J=NY3,NY
      TELX(I,J)=-H*((35.*(P2+Q2)+9.*R2)*W(I,J)-(104.*Q2+12.*R2)
     . *W(I,J-1)+(114.*Q2+3.*R2)*W(I,J-2)-56.*Q2*W(I,J-3)+
     . 11.*Q2*W(I,J-4)-(104.*P2+12.*R2)*W(I-1,J)+16.*R2*W(I-1,J-1)
     . -4.*R2*W(I-1,J-2)+(114.*P2+3.*R2)*W(I-2,J)-4.*R2*
     . W(I-2,J-1)+R2*W(I-2,J-2)-56.*P2*W(I-3,J)+11.*P2*W(I-4,J))
      TELY(I,J)=-H*((35.*(P3+Q3)+9.*R3)*W(I,J)-(104.*Q3+12.*R3)
     . *W(I,J-1)+(114.*Q3+3.*R3)*W(I,J-2)-56.*Q3*W(I,J-3)+
```

```
     • 11.*Q3*W(I,J-4)-(104.*P3+12.*R3)*W(I-1,J)+16.*R3*W(I-1,J-1)
     • -4.*R3*W(I-1,J-2)+(114.*P3+3.*R3)*W(I-2,J)-4.*R3*
     • W(I-2,J-1)+R3*W(I-2,J-2)-56.*P3*W(I-3,J)+11.*P3*W(I-4,J))
       TELXY(I,J)=-H*((35.*(P4+Q4)+9.*R4)*W(I,J)-(104.*Q4+12.*R4)
     • *W(I,J-1)+(114.*Q4+3.*R4)*W(I,J-2)-56.*Q4*W(I,J-3)+
     • 11.*Q4*W(I,J-4)-(104.*P4+12.*R4)*W(I-1,J)+16.*R4*W(I-1,J-1)
     • -4.*R4*W(I-1,J-2)+(114.*P4+3.*R4)*W(I-2,J)-4.*R4*
     • W(I-2,J-1)+R4*W(I-2,J-2)-56.*P4*W(I-3,J)+11.*P4*W(I-4,J))
820    CONTINUE
       DO 830 I=NX3,NX
       DO 830 J=1,NY4
       TELX(I,J)=-H*((35.*(P2+Q2)-9.*R2)*W(I,J)-(104.*Q2-12.*R2)
     • *W(I,J+1)+(114.*Q2-3.*R2)*W(I,J+2)-56.*Q2*W(I,J+3)+
     • 11.*Q2*W(I,J+4)-(104.*P2-12.*R2)*W(I-1,J)-16.*R2*W(I-1,J+1)
     • +4.*R2*W(I-1,J+2)+(114.*P2-3.*R2)*W(I-2,J)+4.*R2*
     • W(I-2,J+1)-R2*W(I-2,J+2)-56.*P2*W(I-3,J)+11.*P2*W(I-4,J))
       TELY(I,J)=-H*((35.*(P3+Q3)-9.*R3)*W(I,J)-(104.*Q3-12.*R3)
     • *W(I,J+1)+(114.*Q3-3.*R3)*W(I,J+2)-56.*Q3*W(I,J+3)+
     • 11.*Q3*W(I,J+4)-(104.*P3-12.*R3)*W(I-1,J)-16.*R3*W(I-1,J+1)
     • +4.*R3*W(I-1,J+2)+(114.*P3-3.*R3)*W(I-2,J)+4.*R3*
     • W(I-2,J+1)-R3*W(I-2,J+2)-56.*P3*W(I-3,J)+11.*P3*W(I-4,J))
       TELXY(I,J)=-H*((35.*(P4+Q4)-9.*R4)*W(I,J)-(104.*Q4-12.*R4)
     • *W(I,J+1)+(114.*Q4-3.*R4)*W(I,J+2)-56.*Q4*W(I,J+3)+
     • 11.*Q4*W(I,J+4)-(104.*P4-12.*R4)*W(I-1,J)-16.*R4*W(I-1,J+1)
     • +4.*R4*W(I-1,J+2)+(114.*P4-3.*R4)*W(I-2,J)+4.*R4*
     • W(I-2,J+1)-R4*W(I-2,J+2)-56.*P4*W(I-3,J)+11.*P4*W(I-4,J))
830    CONTINUE
       DO 840 I=5,NX4
       DO 840 J=5,NY4
       TELX(I,J)=-H*(-P2*W(I-2,J)+16.*P2*W(I-1,J)+R2*(W(I-1,J-1)-
     • W(I-1,J+1))-30.*(P2+Q2)*W(I,J)-Q2*W(I,J-2)+16.*Q2*W(I,J-1)-
     • R2*(W(I+1,J-1)-W(I+1,J+1))+16.*(P2*W(I+1,J)+Q2*W(I,J+1))
```

```
     . -P2*W(I+2,J)-Q2*W(I,J+2))
      TELY(I,J)=-H*(-P3*W(I-2,J)+16.*P3*W(I-1,J)+R3*(W(I-1,J-1)-
     . W(I-1,J+1))-30.*(P3+Q3)*W(I,J)-Q3*W(I,J-2)+16.*Q3*W(I,J-1)-
     . R3*(W(I+1,J-1)-W(I+1,J+1))+16.*(P3*W(I+1,J)+Q3*W(I,J+1))
     . -P3*W(I+2,J)-Q3*W(I,J+2))
      TELXY(I,J)=-H*(-P4*W(I-2,J)+16.*P4*W(I-1,J)+R4*(W(I-1,J-1)-
     . W(I-1,J+1))-30.*(P4+Q4)*W(I,J)-Q4*W(I,J-2)+16.*Q4*W(I,J-1)-
     . R4*(W(I+1,J-1)-W(I+1,J+1))+16.*(P4*W(I+1,J)+Q4*W(I,J+1))
     . -P4*W(I+2,J)-Q4*W(I,J+2))
  840 CONTINUE
      ALFA1H=H*ALFA(1)
      ALFA2H=H*ALFA(2)
      ALFA6H=H*ALFA(6)
      DO 50 I=1,NX
      DO 50 J=1,NY
      TELX(I,J)=TELX(I,J)-ALFA1H*T1(I,J)
      TELY(I,J)=TELY(I,J)-ALFA2H*T1(I,J)
   50 TELXY(I,J)=TELXY(I,J)-ALFA6H*T1(I,J)
C
C        PRINT STRESS DISTRIBUTION TAU-1 ON THE UPPER SURFACE
C
      WRITE (6,55)  (J,J=1,6)
      DO 60 I=1,NX
   60 WRITE (6,1100)  I,(TELX(I,J),J=1,NY)
      WRITE (6,65)  (J,J=1,6)
      DO 70 I=1,NX
   70 WRITE (6,1100)  I,(TELY(I,J),J=1,NY)
      WRITE (6,75)  (J,J=1,6)
      DO 80 I=1,NX
   80 WRITE (6,1100)  I,(TELXY(I,J),J=1,NY)
   55 FORMAT(1H1//////////1X,T35,'STRESS SIGMAX-1 ON THE UPPER SURFACE (
     .PSI)'///17X,6(I2,12X)/)
```

```
 65  FORMAT(1H1//////////1X,T35,'STRESS SIGMAY-1 ON THE UPPER SURFACE (
    .PSI)'///17X,6(I2,12X)/)
 75  FORMAT(1H1//////////1X,T35,'STRESS SIGMAXY-1 ON THE UPPER SURFACE
    .(PSI)'///17X,6(I2,12X)/)
1100 FORMAT (1H,I12,(T15,6E14.5))
     RETURN
     END


C                         INPUT DATA

C        NX,NY,DELH,DELK,H

   15    11    .4      .4      .1

C        ELASTIC CONSTANTS (COMPLIANCES) A(I,J)*1.E-8

   2.        -5.       -4.5       0.        0.        -7.
  -5.         5.       -2.2       0.        0.        10.
  -4.5       -2.2      10.2       0.        0.         9.
   0.         0.        0.       12.       13.75       0.
   0.         0.        0.       13.75     11.         0.
  -7.        10.        9.        0.        0.        10.

C        LINEAR COEFFICIENTS OF THERMAL EXPANSION (ALFA*1.E-6)

  12.5       10.        9.        9.5

C        POLYNOMIAL COEFFICIENTS U1,U2,....U15, AND V1,V2,...V15
```

0.	0.	0.	0.	0.	2.	0.	1.
0.	0.	0.	.6	0.	0.	100.	
0.	1.	0.	0.	0.	0.	0.	3.
0.	0.	0.	0.	.5	0.	400.	
0.	0.	0.	0.	0.	2.	0.	1.
0.	0.	0.	.6	0.	0.	100.	
0.	1.	0.	0.	0.	0.	0.	3.
0.	0.	0.	0.	.5	0.	400.	

APPENDIX B

RESULTS

TO TEMPERATURE DISTRIBUTION (DEGREES C)

	1	2	3	4	5	6
1	0.1000E+03	0.1000E+03	0.1003E+03	0.1010E+03	0.1025E+03	0.1048E+03
	0.1083E+03	0.1132E+03	0.1197E+03	0.1280E+03	0.1384E+03	
2	0.1003E+03	0.1005E+03	0.1009E+03	0.1018E+03	0.1034E+03	0.1059E+03
	0.1096E+03	0.1146E+03	0.1213E+03	0.1298E+03	0.1403E+03	
3	0.1013E+03	0.1016E+03	0.1022E+03	0.1033E+03	0.1050E+03	0.1077E+03
	0.1115E+03	0.1167E+03	0.1235E+03	0.1322E+03	0.1429E+03	
4	0.1029E+03	0.1034E+03	0.1041E+03	0.1054E+03	0.1073E+03	0.1101E+03
	0.1141E+03	0.1194E+03	0.1264E+03	0.1352E+03	0.1461E+03	
5	0.1051E+03	0.1058E+03	0.1067E+03	0.1081E+03	0.1101E+03	0.1131E+03
	0.1173E+03	0.1228E+03	0.1299E+03	0.1389E+03	0.1499E+03	
6	0.1080E+03	0.1088E+03	0.1099E+03	0.1114E+03	0.1137E+03	0.1168E+03
	0.1211E+03	0.1268E+03	0.1341E+03	0.1432E+03	0.1544E+03	
7	0.1115E+03	0.1125E+03	0.1137E+03	0.1154E+03	0.1178E+03	0.1211E+03
	0.1256E+03	0.1314E+03	0.1389E+03	0.1482E+03	0.1595E+03	
8	0.1157E+03	0.1168E+03	0.1182E+03	0.1201E+03	0.1226E+03	0.1261E+03
	0.1307E+03	0.1367E+03	0.1443E+03	0.1538E+03	0.1653E+03	
9	0.1205E+03	0.1218E+03	0.1233E+03	0.1254E+03	0.1281E+03	0.1317E+03
	0.1365E+03	0.1426E+03	0.1504E+03	0.1600E+03	0.1717E+03	
10	0.1259E+03	0.1274E+03	0.1291E+03	0.1313E+03	0.1341E+03	0.1379E+03
	0.1429E+03	0.1492E+03	0.1571E+03	0.1669E+03	0.1787E+03	
11	0.1320E+03	0.1336E+03	0.1355E+03	0.1378E+03	0.1409E+03	0.1448E+03
	0.1499E+03	0.1564E+03	0.1645E+03	0.1744E+03	0.1864E+03	
12	0.1387E+03	0.1405E+03	0.1425E+03	0.1450E+03	0.1482E+03	0.1523E+03
	0.1576E+03	0.1642E+03	0.1725E+03	0.1826E+03	0.1947E+03	
13	0.1461E+03	0.1480E+03	0.1502E+03	0.1529E+03	0.1562E+03	0.1605E+03
	0.1659E+03	0.1727E+03	0.1811E+03	0.1914E+03	0.2037E+03	
14	0.1541E+03	0.1562E+03	0.1585E+03	0.1614E+03	0.1649E+03	0.1693E+03
	0.1749E+03	0.1818E+03	0.1904E+03	0.2008E+03	0.2133E+03	
15	0.1627E+03	0.1650E+03	0.1675E+03	0.1705E+03	0.1741E+03	0.1787E+03
	0.1845E+03	0.1916E+03	0.2003E+03	0.2109E+03	0.2235E+03	

TEMPERATURE DISTRIBUTION DUE TO ZT1 ON THE SURFACE (DEGREES C)

	1	2	3	4	5	6
1	0.4000E+02	0.4000E+02	0.4000E+02	0.4000E+02	0.4000E+02	0.4000E+02
	0.4000E+02	0.4000E+02	0.4000E+02	0.4000E+02	0.4000E+02	
2	0.4000E+02	0.4005E+02	0.4011E+02	0.4019E+02	0.4028E+02	0.4041E+02
	0.4058E+02	0.4079E+02	0.4106E+02	0.4139E+02	0.4179E+02	
3	0.4000E+02	0.4012E+02	0.4025E+02	0.4042E+02	0.4063E+02	0.4090E+02
	0.4125E+02	0.4169E+02	0.4224E+02	0.4291E+02	0.4372E+02	
4	0.4000E+02	0.4022E+02	0.4046E+02	0.4074E+02	0.4110E+02	0.4155E+02
	0.4211E+02	0.4281E+02	0.4367E+02	0.4472E+02	0.4597E+02	
5	0.4000E+02	0.4036E+02	0.4075E+02	0.4121E+02	0.4175E+02	0.4242E+02
	0.4324E+02	0.4425E+02	0.4547E+02	0.4694E+02	0.4868E+02	
6	0.4000E+02	0.4057E+02	0.4117E+02	0.4185E+02	0.4265E+02	0.4360E+02
	0.4474E+02	0.4612E+02	0.4776E+02	0.4971E+02	0.5200E+02	
7	0.4000E+02	0.4085E+02	0.4174E+02	0.4273E+02	0.4386E+02	0.4516E+02
	0.4670E+02	0.4852E+02	0.5066E+02	0.5317E+02	0.5609E+02	
8	0.4000E+02	0.4122E+02	0.4250E+02	0.4388E+02	0.4543E+02	0.4719E+02
	0.4922E+02	0.5157E+02	0.5430E+02	0.5746E+02	0.6110E+02	
9	0.4000E+02	0.4170E+02	0.4347E+02	0.4536E+02	0.4743E+02	0.4975E+02
	0.5238E+02	0.5538E+02	0.5880E+02	0.6272E+02	0.6719E+02	
10	0.4000E+02	0.4231E+02	0.4469E+02	0.4721E+02	0.4993E+02	0.5293E+02
	0.5628E+02	0.6004E+02	0.6428E+02	0.6908E+02	0.7450E+02	
11	0.4000E+02	0.4305E+02	0.4618E+02	0.4947E+02	0.5298E+02	0.5680E+02
	0.6100E+02	0.6567E+02	0.7087E+02	0.7669E+02	0.8320E+02	
12	0.4000E+02	0.4395E+02	0.4798E+02	0.5219E+02	0.5664E+02	0.6144E+02
	0.6665E+02	0.7238E+02	0.7869E+02	0.8568E+02	0.9343E+02	
13	0.4000E+02	0.4502E+02	0.5012E+02	0.5541E+02	0.6098E+02	0.6692E+02
	0.7332E+02	0.8027E+02	0.8786E+02	0.9619E+02	0.1054E+03	
14	0.4000E+02	0.4626E+02	0.5263E+02	0.5919E+02	0.6606E+02	0.7332E+02
	0.8108E+02	0.8945E+02	0.9851E+02	0.1084E+03	0.1191E+03	
15	0.4000E+02	0.4771E+02	0.5554E+02	0.6357E+02	0.7193E+02	0.8072E+02
	0.9005E+02	0.1000E+03	0.1107E+03	0.1223E+03	0.1349E+03	

STRESS SIGMAX-0 (PSI)

	1	2	3	4	5	6
1	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
2	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
3	0.99754E+00	-0.25646E+03	0.44638E+03	-0.64536E+03	0.44638E+03	-0.64536E+03
	0.57660E+03	-0.26318E+03	0.34983E+03	0.11925E+03	-0.32806E+03	
4	-0.37022E+03	0.58296E+03	-0.83500E+03	0.89916E+03	-0.83500E+03	0.89916E+03
	-0.97755E+03	0.10938E+04	-0.56630E+02	0.13727E+03	-0.47381E+03	
5	-0.14631E+03	-0.53939E+03	0.10679E+04	-0.13312E+04	-0.35468E+03	0.40737E+03
	-0.28707E+03	-0.10401E+04	0.10867E+04	-0.16457E+03	-0.50961E+03	
6	-0.54690E+03	0.72972E+03	-0.98931E+03	0.14781E+04	0.39141E+03	-0.47894E+03
	0.51682E+03	0.20272E+04	-0.82560E+03	0.59091E+03	-0.66192E+03	
7	-0.18633E+03	-0.55495E+03	0.12974E+04	-0.15716E+04	-0.41974E+03	0.49752E+03
	-0.50787E+03	-0.18937E+04	0.20754E+04	-0.55985E+03	-0.49110E+03	
8	-0.40661E+03	0.52054E+03	-0.91910E+03	0.15003E+04	0.39854E+03	-0.52922E+03
	0.61937E+03	0.24093E+04	-0.14426E+04	0.12344E+04	-0.90754E+03	
9	-0.16284E+03	-0.45177E+03	0.99114E+03	-0.12497E+04	-0.33560E+03	0.43955E+03
	-0.54355E+03	-0.20285E+04	0.25651E+04	-0.87680E+03	-0.43635E+03	
10	-0.28313E+03	0.21947E+03	-0.59347E+03	0.11357E+04	0.29915E+03	-0.38948E+03
	0.48339E+03	0.18643E+04	-0.14477E+04	0.17330E+04	-0.11733E+04	
11	-0.18742E+03	-0.21826E+03	0.47649E+03	-0.67632E+03	-0.18381E+03	0.28690E+03
	-0.36979E+03	-0.13654E+04	0.19876E+04	-0.66395E+03	-0.43643E+03	
12	-0.20213E+03	0.98263E+01	-0.18920E+03	0.57175E+03	-0.58898E+03	0.90553E+03
	-0.96427E+03	0.90553E+03	-0.96427E+03	0.13926E+04	-0.93512E+03	
13	-0.15147E+03	-0.66679E+01	0.14900E+03	-0.16228E+03	0.39175E+03	-0.46261E+03
	0.63856E+03	-0.46261E+03	0.63856E+03	-0.28049E+03	-0.11580E+03	
14	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
15	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	

STRESS SIGMAY-0 (PSI)

	1	2	3	4	5	6
1	0.0	0.0	0.99754E+00	-0.25447E+03	-0.63553E+02	-0.51800E+03
	-0.39584E+03	-0.53687E+03	-0.32806E+03	0.0	0.0	
2	0.0	0.0	-0.37222E+03	0.35145E+03	-0.65264E+03	0.53314E+03
	-0.41183E+03	0.26339E+03	0.18231E+03	0.0	0.0	
3	0.0	0.0	0.59513E+03	-0.77151E+03	0.10461E+04	-0.91108E+03
	0.12399E+04	-0.99968E+02	0.10995E+03	0.0	0.0	
4	0.0	0.0	-0.62449E+03	0.11425E+04	-0.10507E+04	0.17958E+04
	-0.12906E+04	0.82429E+03	-0.11652E+03	0.0	0.0	
5	0.0	0.0	0.76115E+03	-0.10315E+04	-0.28792E+03	0.46399E+03
	-0.35463E+03	-0.99968E+02	0.10995E+03	0.0	0.0	
6	0.0	0.0	-0.58085E+03	0.11985E+04	0.39756E+03	-0.47828E+03
	0.51941E+03	0.82429E+03	-0.11652E+03	0.0	0.0	
7	0.0	0.0	0.46405E+03	-0.11197E+04	-0.40500E+03	0.49756E+03
	-0.60695E+03	-0.12600E+04	0.32314E+03	0.0	0.0	
8	0.0	0.0	-0.36406E+03	0.91542E+03	0.37919E+03	-0.49118E+03
	0.62965E+03	0.17705E+04	-0.58726E+03	0.0	0.0	
9	0.0	0.0	0.21601E+03	-0.67696E+03	-0.34512E+03	0.43147E+03
	-0.58457E+03	-0.21302E+04	0.88763E+03	0.0	0.0	
10	0.0	0.0	-0.11043E+03	0.44497E+03	0.28701E+03	-0.36136E+03
	0.48960E+03	0.23047E+04	-0.12081E+04	0.0	0.0	
11	0.0	0.0	0.65381E+02	-0.11382E+03	-0.20149E+03	0.29703E+03
	-0.32537E+03	-0.20591E+04	0.14738E+04	0.0	0.0	
12	0.0	0.0	-0.11043E+03	0.44497E+03	-0.73529E+03	0.11446E+04
	-0.11869E+04	0.19823E+04	-0.12356E+04	0.0	0.0	
13	0.0	0.0	0.65381E+02	-0.11382E+03	0.71087E+03	-0.44654E+03
	0.10495E+04	-0.10936E+04	0.13180E+04	0.0	0.0	
14	0.0	0.0	0.10080E+03	0.22476E+03	-0.13848E+03	0.39460E+03
	-0.44481E+03	0.54653E+03	-0.70352E+03	0.0	0.0	
15	0.0	0.0	-0.15147E+03	-0.30960E+03	-0.31874E+03	-0.49015E+03
	-0.26982E+03	-0.51209E+03	-0.11580E+03	0.0	0.0	

STRESS SIGMAXY-0 (PSI)

	1	2	3	4	5	6
1	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	
2	0.0	0.99754E+00	-0.25546E+03	0.19091E+03	0.19091E+03	-0.45444E+03
	0.12216E+03	-0.14103E+03	0.20881E+03	0.32806E+03	0.0	
3	0.0	-0.37122E+03	0.46820E+03	-0.81318E+03	-0.81318E+03	0.73134E+03
	-0.82281E+03	0.53419E+03	0.12773E+03	0.14575E+03	0.0	
4	0.0	0.22391E+03	-0.89844E+03	0.10045E+04	0.10045E+04	-0.12259E+04
	0.13282E+04	-0.80570E+03	0.33764E+03	0.35801E+02	0.0	
5	0.0	-0.40058E+03	0.86853E+03	-0.11887E+04	0.10524E+02	-0.17621E+01
	0.36749E+01	-0.80570E+03	0.33764E+03	0.35801E+02	0.0	
6	0.0	0.36057E+03	-0.92411E+03	0.13626E+04	0.53743E+01	0.87639E+01
	-0.18508E+02	0.13091E+04	-0.60316E+03	0.15232E+03	0.0	
7	0.0	-0.22028E+03	0.85522E+03	-0.13613E+04	0.12436E+01	-0.69752E+01
	0.28006E+01	-0.19211E+04	0.97994E+03	-0.17082E+03	0.0	
8	0.0	0.24377E+03	-0.72854E+03	0.11817E+04	-0.18654E+01	0.24766E+01
	-0.16014E+02	0.21401E+04	-0.13778E+04	0.41644E+03	0.0	
9	0.0	-0.12029E+03	0.55094E+03	-0.10337E+04	-0.34262E+01	0.50397E+01
	0.44846E+01	-0.23676E+04	0.16400E+04	-0.47119E+03	0.0	
10	0.0	0.95712E+02	-0.34202E+03	0.72794E+03	-0.19051E+01	-0.15332E+01
	0.33225E+01	0.21400E+04	-0.18728E+04	0.73695E+03	0.0	
11	0.0	-0.14713E+02	0.21338E+03	-0.45232E+03	-0.63702E+00	0.14461E+02
	0.98174E+01	-0.17752E+04	0.16601E+04	-0.73687E+03	0.0	
12	0.0	-0.14713E+02	0.21338E+03	-0.45232E+03	0.79575E+03	-0.87695E+03
	0.13940E+04	0.13940E+04	-0.15578E+04	0.49870E+03	0.0	
13	0.0	0.50668E+02	0.34174E+02	0.37238E+03	-0.36166E+03	0.61907E+03
	-0.74907E+03	-0.74907E+03	0.85376E+03	-0.81932E+03	0.0	
14	0.0	0.15147E+03	0.15813E+03	0.91367E+01	0.17142E+03	-0.22034E+03
	0.24227E+03	0.24227E+03	-0.39629E+03	-0.11580E+03	0.0	
15	0.0	0.0	0.0	0.0	0.0	0.0
	0.0	0.0	0.0	0.0	0.0	

DEFLECTION (INCH)

	1	2	3	4	5	6
1	-0.3923E-02	-0.1137E-01	-0.1878E-01	-0.1886E-01	-0.1610E-01	-0.3522E-01
	-0.8289E-02	-0.9520E-02	0.1319E-02	-0.7740E-03	-0.4542E-03	
2	0.9924E-02	0.7324E-03	0.1875E-02	0.1189E-02	0.3851E-02	0.1407E-02
	0.3028E-02	0.3366E-04	0.1054E-02	-0.4684E-04	0.2669E-03	
3	0.1631E-01	0.1820E-02	-0.1082E-02	-0.7828E-03	-0.9031E-03	-0.1129E-02
	-0.8721E-03	-0.7408E-04	-0.2369E-04	-0.1476E-03	-0.3404E-03	
4	0.1472E-01	0.1517E-02	0.1858E-03	0.2287E-03	0.3036E-03	0.3287E-03
	0.2571E-03	0.2056E-03	0.1003E-04	-0.8834E-04	-0.2904E-03	
5	0.1471E-01	0.1516E-02	-0.1989E-03	-0.6349E-04	-0.3765E-04	-0.4146E-04
	-0.4048E-04	-0.1577E-04	-0.1071E-04	-0.1102E-03	-0.3073E-03	
6	0.1469E-01	0.1514E-02	-0.1477E-03	-0.2748E-04	0.1097E-05	-0.1610E-05
	-0.8760E-06	0.4645E-05	-0.1337E-04	-0.1398E-03	-0.3259E-03	
7	0.1467E-01	0.1512E-02	-0.1473E-03	-0.2737E-04	0.1046E-05	-0.1768E-05
	-0.7599E-06	0.4360E-05	-0.1497E-04	-0.1755E-03	-0.3488E-03	
8	0.1463E-01	0.2572E-02	-0.1301E-03	-0.3041E-04	0.1015E-05	-0.1942E-05
	-0.5834E-06	-0.3737E-05	0.2181E-04	0.7209E-03	-0.3767E-03	
9	0.9521E-02	-0.3709E-02	-0.2313E-03	-0.3028E-05	0.1015E-05	-0.2027E-05
	-0.4366E-06	0.6026E-04	-0.2027E-03	-0.4348E-02	0.4329E-05	
10	0.4156E-01	0.9738E-02	-0.1277E-03	-0.5392E-04	0.2619E-05	-0.7882E-06
	0.1422E-05	-0.8433E-04	0.1705E-03	0.4009E-02	0.1934E-02	
11	-0.6397E-02	-0.1535E-02	-0.1815E-03	-0.6359E-04	-0.2989E-04	-0.3448E-04
	-0.4874E-04	-0.1606E-03	0.1530E-03	0.1183E-01	-0.3044E-02	
12	0.2172E-02	-0.2373E-02	0.1469E-03	0.2149E-03	0.2144E-03	0.2350E-04
	0.2535E-03	0.4868E-03	-0.8950E-03	-0.1305E-01	-0.7513E-02	
13	0.5965E-02	-0.7227E-02	0.5091E-04	-0.2869E-04	-0.1503E-03	0.3797E-03
	0.1543E-03	0.4529E-03	-0.1070E-02	0.3084E-02	0.1165E-01	
14	-0.2390E-02	0.1653E-02	-0.5041E-04	0.8774E-03	-0.1592E-02	0.5425E-03
	-0.6107E-02	0.4016E-02	-0.1141E-02	-0.1061E-01	-0.3689E-01	
15	0.4199E-03	-0.3556E-02	-0.2231E-02	-0.2537E-02	-0.4217E-02	0.1216E-02
	-0.5153E-02	0.5683E-02	-0.2636E-01	0.1215E-01	-0.1085E-01	

STRESS SIGMAX-1 ON THE UPPER SURFACE (PSI)

	1	2	3	4	5	6
1	0.37353E+06	-0.93206E+06	-0.37420E+07	0.53527E+06	-0.25403E+07	-0.33015E+07
	-0.35221E+07	0.18032E+07	-0.18433E+07	0.10229E+07	-0.79931E+06	
2	-0.11745E+07	0.20865E+06	0.16847E+06	0.83808E+06	0.45601E+06	0.78401E+06
	0.27991E+06	0.36959E+06	-0.16830E+06	0.27924E+06	-0.60414E+05	
3	-0.14348E+06	-0.11011E+06	-0.14466E+06	-0.10630E+06	-0.15343E+06	-0.21918E+06
	-0.13569E+06	-0.25815E+05	0.58318E+05	-0.86936E+04	0.25329E+05	
4	-0.34529E+06	0.15549E+05	0.62620E+05	0.54744E+05	-0.21952E+05	0.51758E+05
	0.57341E+05	0.43414E+05	0.31620E+05	0.44659E+05	0.35895E+05	
5	-0.44958E+06	-0.33459E+06	0.18280E+05	0.23392E+05	0.42455E+05	0.43945E+05
	0.40451E+05	-0.31975E+06	-0.95450E+05	-0.10392E+05	-0.68640E+05	
6	0.11360E+07	0.12020E+07	0.43489E+05	0.21110E+05	0.23436E+05	0.23470E+05
	0.24382E+05	0.10228E+06	0.67373E+05	0.42967E+05	0.52668E+05	
7	-0.61214E+07	-0.29536E+07	-0.35621E+04	0.36725E+05	0.26542E+05	0.26901E+05
	0.27841E+05	-0.67614E+04	0.28094E+05	0.34151E+05	0.26952E+05	
8	0.11277E+08	0.37948E+07	0.64022E+05	0.26001E+05	0.27508E+05	0.27990E+05
	0.29262E+05	0.39203E+05	0.33896E+05	0.75834E+05	0.14551E+06	
9	-0.12000E+08	-0.26631E+07	-0.20308E+05	-0.94079E+04	0.28200E+05	0.29734E+05
	0.30412E+05	0.36157E+05	0.28836E+05	-0.33496E+06	-0.49672E+06	
10	0.38476E+07	0.10822E+07	0.92648E+05	0.13236E+06	0.28604E+05	0.29877E+05
	0.32800E+05	0.18610E+05	0.84914E+05	0.14343E+07	0.69651E+06	
11	0.11474E+07	-0.18167E+07	-0.63829E+05	-0.23160E+06	0.43726E+05	0.36249E+05
	0.50219E+05	0.67489E+05	-0.64774E+05	-0.17795E+07	0.88031E+06	
12	0.12780E+08	0.36909E+07	0.73086E+05	0.70521E+05	0.69198E+05	0.10781E+06
	-0.44415E+06	0.10516E+06	0.95065E+05	-0.15378E+07	-0.90828E+06	
13	-0.81582E+07	-0.17179E+07	-0.99647E+04	-0.81878E+05	0.36132E+04	-0.74086E+05
	0.18807E+06	-0.12389E+06	0.28464E+06	0.56125E+07	0.21793E+07	
14	0.76467E+06	0.18064E+07	0.40952E+06	-0.39677E+06	0.97282E+06	-0.85492E+06
	-0.95451E+05	-0.72513E+06	0.12957E+07	-0.70654E+07	-0.65875E+07	
15	0.19796E+07	-0.26612E+07	0.29991E+06	-0.12943E+07	0.14330E+07	-0.27625E+07
	0.56079E+07	-0.14325E+07	0.59477E+06	0.15606E+07	0.17944E+08	

STRESS SIGMAY-1 ON THE UPPER SURFACE (PSI)

	1	2	3	4	5	6
1	-0.27213E+06	0.12712E+05	0.29479E+07	-0.28284E+07	0.83921E+06	0.77111E+05
	0.47547E+07	-0.37667E+07	0.29523E+07	-0.17276E+07	0.10168E+07	
2	0.10790E+07	-0.17755E+06	0.23632E+06	-0.60706E+06	0.22395E+06	-0.71476E+06
	0.32196E+06	-0.63376E+06	0.49832E+06	-0.40831E+06	0.23171E+06	
3	0.35268E+06	0.13501E+06	0.72985E+05	0.35486E+05	0.68123E+05	0.14133E+06
	0.94491E+05	0.44631E+05	-0.51986E+05	0.37999E+05	-0.24100E+04	
4	0.52978E+06	0.65478E+05	-0.15264E+05	-0.76776E+04	0.11100E+06	0.39258E+04
	-0.15018E+05	-0.36038E+04	-0.12736E+05	0.50896E+04	-0.10498E+05	
5	0.54277E+06	0.21438E+06	0.15043E+04	0.97663E+03	-0.55429E+04	-0.59253E+04
	-0.42804E+04	0.13207E+06	0.44658E+05	0.17196E+05	0.40500E+05	
6	-0.74891E+05	-0.41149E+06	-0.84115E+04	0.19400E+04	0.20295E+04	0.27285E+04
	0.27565E+04	-0.27454E+05	-0.12636E+05	-0.66187E+04	-0.58205E+04	
7	0.27439E+07	0.13472E+07	0.12656E+05	-0.44576E+04	0.11234E+04	0.17963E+04
	0.18696E+04	0.15597E+05	0.16377E+04	-0.24461E+04	0.84720E+04	
8	-0.43004E+07	-0.16996E+07	-0.20577E+05	0.15247E+04	0.11862E+04	0.19398E+04
	0.16135E+04	-0.19177E+04	0.13436E+04	-0.15046E+05	-0.96078E+05	
9	0.62392E+07	0.14450E+07	0.17582E+05	0.14856E+05	0.18690E+04	0.18600E+04
	0.37516E+04	0.86844E+03	0.24868E+02	0.16336E+06	0.45081E+06	
10	-0.20981E+07	-0.36293E+06	-0.24975E+05	-0.42406E+05	0.13987E+04	0.29586E+04
	0.13687E+04	0.97676E+04	-0.19454E+05	-0.70129E+06	-0.27476E+06	
11	-0.35346E+06	0.70788E+06	0.36543E+05	0.99957E+05	-0.25547E+04	0.10625E+04
	-0.68034E+04	-0.13307E+05	0.67754E+05	0.98444E+06	-0.13468E+07	
12	-0.41729E+07	-0.14816E+07	-0.53363E+04	-0.36603E+05	-0.77468E+04	-0.95955E+05
	0.69530E+06	-0.46651E+05	-0.18924E+05	0.10311E+07	0.45486E+06	
13	0.44767E+07	0.31618E+06	-0.12787E+05	0.11500E+06	-0.78627E+05	0.21140E+06
	-0.26649E+06	0.12142E+06	-0.19560E+06	-0.26191E+07	-0.57021E+06	
14	-0.13013E+07	-0.13364E+06	-0.58818E+06	0.92387E+06	-0.15574E+07	0.17338E+07
	-0.84588E+06	0.95559E+06	-0.15501E+07	0.39347E+07	0.28623E+07	
15	-0.10184E+06	0.96416E+06	-0.48291E+06	0.13914E+07	-0.21393E+07	0.39894E+07
	-0.71536E+07	0.18221E+07	-0.18171E+07	0.19797E+07	-0.12299E+08	

STRESS SIGMAXY-1 ON THE UPPER SURFACE (PSI)

	1	2	3	4	5	6
1	0.67555E+05	0.50684E+06	-0.30073E+06	0.25275E+07	0.93369E+06	0.20245E+07
	-0.18543E+07	0.24413E+07	-0.15319E+07	0.96850E+06	-0.44567E+06	
2	-0.18696E+06	0.11152E+06	-0.37184E+06	0.81258E+05	-0.48728E+06	0.13346E+06
	-0.47931E+06	0.32282E+06	-0.33786E+06	0.20619E+06	-0.16655E+06	
3	-0.25158E+06	-0.72370E+05	0.44134E+05	0.48041E+05	0.45892E+05	0.20429E+05
	0.17764E+05	-0.46894E+04	0.22826E+05	-0.22198E+05	-0.32808E+04	
4	-0.27930E+06	-0.59446E+05	-0.19291E+05	-0.19116E+05	-0.83413E+05	-0.28801E+05
	-0.14950E+05	-0.16090E+05	-0.19012E+03	-0.23454E+05	-0.22417E+04	
5	-0.21719E+06	0.29897E+05	-0.31119E+04	-0.62986E+04	-0.12827E+05	-0.13493E+05
	-0.12604E+05	0.99456E+05	0.32101E+05	0.41657E+04	0.20486E+05	
6	-0.70312E+06	-0.42386E+06	-0.11050E+05	-0.54405E+04	-0.69655E+04	-0.74285E+04
	-0.77555E+04	-0.31219E+05	-0.20620E+05	-0.10132E+05	-0.17312E+05	
7	0.15165E+07	0.77468E+06	0.20130E+04	-0.99248E+04	-0.79078E+04	-0.84764E+04
	-0.87799E+04	0.21077E+04	-0.77353E+04	-0.67731E+04	-0.12494E+05	
8	-0.36037E+07	-0.10943E+07	-0.15547E+05	-0.72869E+04	-0.81804E+04	-0.88370E+04
	-0.89423E+04	-0.11534E+05	-0.11137E+05	-0.34614E+05	0.27803E+05	
9	0.25246E+07	0.66982E+06	0.10990E+05	0.30888E+04	-0.88857E+04	-0.92883E+04
	-0.10820E+05	-0.12216E+05	0.18914E+03	0.15872E+06	-0.19005E+06	
10	-0.10936E+07	-0.53524E+06	-0.28765E+05	-0.37100E+05	-0.80316E+04	-0.96044E+04
	-0.88928E+04	-0.42803E+04	-0.32111E+05	-0.41627E+06	-0.31831E+05	
11	-0.34919E+06	0.58132E+06	0.20604E+05	0.73386E+05	-0.13418E+05	-0.11218E+05
	-0.12791E+05	-0.16664E+05	-0.16358E+04	0.20845E+06	0.92714E+06	
12	-0.44700E+07	-0.10543E+07	-0.32129E+05	0.78003E+03	-0.21667E+05	0.35165E+05
	-0.37948E+06	-0.11295E+05	-0.67496E+03	0.43229E+06	-0.26496E+06	
13	0.13428E+07	0.83641E+06	0.34573E+05	-0.37565E+05	0.78977E+05	-0.13422E+06
	0.15725E+06	-0.14916E+05	0.13795E+05	-0.16229E+07	-0.76023E+06	
14	0.44438E+06	-0.96481E+06	0.29000E+06	-0.59136E+06	0.84562E+06	-0.98150E+06
	0.77984E+06	-0.59119E+06	0.76595E+06	0.12914E+07	0.21959E+07	
15	-0.10513E+07	0.81948E+06	0.31737E+06	-0.47745E+06	0.11280E+07	-0.20800E+07
	0.31680E+07	-0.74510E+06	0.17796E+07	-0.38980E+07	-0.18837E+06	

STRESS SIGMAX ON THE UPPER SURFACE (PSI)

	1	2	3	4	5	6
1	0.37353E+06	-0.93206E+06	-0.37420E+07	0.53527E+06	-0.25403E+07	-0.33015E+07
	-0.35221E+07	0.18032E+07	-0.18433E+07	0.10229E+07	-0.79931E+06	
2	-0.11745E+07	0.20865E+06	0.16847E+06	0.83808E+06	0.45601E+06	0.78401E+06
	0.27991E+06	0.36959E+06	-0.16830E+06	0.27924E+06	-0.60414E+05	
3	-0.14348E+06	-0.11016E+06	-0.14457E+06	-0.10643E+06	-0.15334E+06	-0.21931E+06
	-0.13558E+06	-0.25868E+05	0.58388E+05	-0.86698E+04	0.25263E+05	
4	-0.34536E+06	0.15665E+05	0.62453E+05	0.54924E+05	-0.22119E+05	0.51938E+05
	0.57146E+05	0.43633E+05	0.31609E+05	0.44686E+05	0.35800E+05	
5	-0.44961E+06	-0.33470E+06	0.18494E+05	0.23126E+05	0.42384E+05	0.44027E+05
	0.40394E+05	-0.31996E+06	-0.95232E+05	-0.10425E+05	-0.68742E+05	
6	0.11359E+07	0.12021E+07	0.43291E+05	0.21406E+05	0.23514E+05	0.23374E+05
	0.24485E+05	0.10268E+06	0.67207E+05	0.43085E+05	0.52535E+05	
7	-0.61214E+07	-0.29537E+07	-0.33026E+04	0.36411E+05	0.26458E+05	0.27001E+05
	0.27739E+05	-0.71401E+04	0.28509E+05	0.34039E+05	0.26854E+05	
8	0.11277E+08	0.37949E+07	0.63839E+05	0.26301E+05	0.27587E+05	0.27884E+05
	0.29386E+05	0.39685E+05	0.33607E+05	0.76081E+05	0.14533E+06	
9	-0.12000E+08	-0.26632E+07	-0.20110E+05	-0.96578E+04	0.28133E+05	0.29822E+05
	0.30304E+05	0.35752E+05	0.29349E+05	-0.33514E+06	-0.49681E+06	
10	0.38475E+07	0.10822E+07	0.92530E+05	0.13259E+06	0.28664E+05	0.29799E+05
	0.32896E+05	0.18983E+05	0.84625E+05	0.14347E+07	0.69627E+06	
11	0.11474E+07	-0.18168E+07	-0.63734E+05	-0.23174E+06	0.43690E+05	0.36307E+05
	0.50145E+05	0.67216E+05	-0.64377E+05	-0.17796E+07	0.88022E+06	
12	0.12780E+08	0.36909E+07	0.73048E+05	0.70636E+05	0.69081E+05	0.10799E+06
	-0.44434E+06	0.10534E+06	0.94872E+05	-0.15375E+07	-0.90847E+06	
13	-0.81583E+07	-0.17179E+07	-0.99349E+04	-0.81910E+05	0.36915E+04	-0.74178E+05
	0.18820E+06	-0.12399E+06	0.28476E+06	0.56125E+07	0.21793E+07	
14	0.76467E+06	0.18064E+07	0.40952E+06	-0.39677E+06	0.97282E+06	-0.85492E+06
	-0.95451E+05	-0.72513E+06	0.12957E+07	-0.70654E+07	-0.65875E+07	
15	0.19796E+07	-0.26612E+07	0.29991E+06	-0.12943E+07	0.14330E+07	-0.27625E+07
	0.56079E+07	-0.14325E+07	0.59477E+06	0.15606E+07	0.17944E+08	

STRESS SIGMAY ON THE UPPER SURFACE (PSI)

	1	2	3	4	5	6
1	-0.27213E+06	0.12712E+05	0.29479E+07	-0.28284E+07	0.83920E+06	0.77007E+05
	0.47547E+07	-0.37668E+07	0.29523E+07	-0.17276E+07	0.10168E+07	
2	0.10790E+07	-0.17755E+06	0.23624E+06	-0.60699E+06	0.22382E+06	-0.71465E+06
	0.32187E+06	-0.63370E+06	0.49836E+06	-0.40831E+06	0.23171E+06	
3	0.35268E+06	0.13501E+06	0.73104E+05	0.35332E+05	0.68332E+05	0.14115E+06
	0.94739E+05	0.44611E+05	-0.51964E+05	0.37999E+05	-0.24100E+04	
4	0.52978E+06	0.65478E+05	-0.15389E+05	-0.74491E+04	0.11079E+06	0.42849E+04
	-0.15276E+05	-0.34390E+04	-0.12759E+05	0.50896E+04	-0.10498E+05	
5	0.54277E+06	0.21438E+06	0.16565E+04	0.77033E+03	-0.56005E+04	-0.58325E+04
	-0.43514E+04	0.13205E+06	0.44680E+05	0.17196E+05	0.40500E+05	
6	-0.74891E+05	-0.41149E+06	-0.85276E+04	0.21797E+04	0.21090E+04	0.26328E+04
	0.28604E+04	-0.27289E+05	-0.12659E+05	-0.66187E+04	-0.58205E+04	
7	0.27439E+07	0.13472E+07	0.12749E+05	-0.46815E+04	0.10424E+04	0.18958E+04
	0.17482E+04	0.15345E+05	0.17023E+04	-0.24461E+04	0.84720E+04	
8	-0.43004E+07	-0.16996E+07	-0.20650E+05	0.17078E+04	0.12621E+04	0.18416E+04
	0.17394E+04	-0.15636E+04	0.12262E+04	-0.15046E+05	-0.96078E+05	
9	0.62392E+07	0.14450E+07	0.17626E+05	0.14720E+05	0.17999E+04	0.19463E+04
	0.36347E+04	0.44239E+03	0.20239E+03	0.16336E+06	0.45081E+06	
10	-0.20981E+07	-0.36293E+06	-0.24997E+05	-0.42317E+05	0.14561E+04	0.28863E+04
	0.14666E+04	0.10229E+05	-0.19695E+05	-0.70129E+06	-0.27476E+06	
11	-0.35346E+06	0.70788E+06	0.36556E+05	0.99934E+05	-0.25950E+04	0.11219E+04
	-0.68684E+04	-0.13719E+05	0.68049E+05	0.98444E+06	-0.13468E+07	
12	-0.41729E+07	-0.14816E+07	-0.53583E+04	-0.36514E+05	-0.78938E+04	-0.95726E+05
	0.69506E+06	-0.46255E+05	-0.19171E+05	0.10311E+07	0.45486E+06	
13	0.44767E+07	0.31618E+06	-0.12774E+05	0.11498E+06	-0.78485E+05	0.21131E+06
	-0.26628E+06	0.12120E+06	-0.19534E+06	-0.26191E+07	-0.57021E+06	
14	-0.13013E+07	-0.13364E+06	-0.58816E+06	0.92391E+06	-0.15574E+07	0.17339E+07
	-0.84596E+06	0.95570E+06	-0.15502E+07	0.39347E+07	0.28623E+07	
15	-0.10184E+06	0.96416E+06	-0.48294E+06	0.13913E+07	-0.21393E+07	0.39893E+07
	-0.71536E+07	0.18220E+07	-0.18171E+07	0.19797E+07	-0.12299E+08	

STRESS SIGMAXY ON THE UPPER SURFACE (PSI)

	1	2	3	4	5	6
1	0.67555E+05	0.50684E+06	-0.30073E+06	0.25275E+07	0.93369E+06	0.20245E+07
	-0.18543E+07	0.24413E+07	-0.15319E+07	0.96850E+06	-0.44567E+06	
2	-0.18696E+06	0.11152E+06	-0.37189E+06	0.81297E+05	-0.48724E+06	0.13337E+06
	-0.47928E+06	0.32279E+06	-0.33782E+06	0.20626E+06	-0.16655E+06	
3	-0.25158E+06	-0.72444E+05	0.44227E+05	0.47878E+05	0.45729E+05	0.20575E+05
	0.17600E+05	-0.45825E+04	0.22852E+05	-0.22169E+05	-0.32808E+04	
4	-0.27930E+06	-0.59401E+05	-0.19470E+05	-0.18916E+05	-0.83212E+05	-0.29046E+05
	-0.14685E+05	-0.16252E+05	-0.12259E+03	-0.23447E+05	-0.22417E+04	
5	-0.21719E+06	0.29817E+05	-0.29382E+04	-0.65364E+04	-0.12825E+05	-0.13494E+05
	-0.12603E+05	0.99295E+05	0.32169E+05	0.41729E+04	0.20486E+05	
6	-0.70312E+06	-0.42379E+06	-0.11235E+05	-0.51680E+04	-0.69645E+04	-0.74268E+04
	-0.77592E+04	-0.30957E+05	-0.20741E+05	-0.10102E+05	-0.17312E+05	
7	0.15165E+07	0.77464E+06	0.21840E+04	-0.10197E+05	-0.79075E+04	-0.84778E+04
	-0.87793E+04	0.17235E+04	-0.75393E+04	-0.68073E+04	-0.12494E+05	
8	-0.36037E+07	-0.10942E+07	-0.15692E+05	-0.70505E+04	-0.81808E+04	-0.88365E+04
	-0.89455E+04	-0.11106E+05	-0.11412E+05	-0.34531E+05	0.27803E+05	
9	0.25246E+07	0.66979E+06	0.11101E+05	0.28821E+04	-0.88864E+04	-0.92873E+04
	-0.10819E+05	-0.12690E+05	0.51715E+03	0.15862E+06	-0.19005E+06	
10	-0.10936E+07	-0.53522E+06	-0.28833E+05	-0.36954E+05	-0.80320E+04	-0.96047E+04
	-0.88921E+04	-0.38523E+04	-0.32485E+05	-0.41612E+06	-0.31831E+05	
11	-0.34919E+06	0.58131E+06	0.20646E+05	0.73296E+05	-0.13418E+05	-0.11215E+05
	-0.12789E+05	-0.17020E+05	-0.13038E+04	0.20830E+06	0.92714E+06	
12	-0.44700E+07	-0.10543E+07	-0.32086E+05	0.68957E+03	-0.21508E+05	0.34990E+05
	-0.37920E+06	-0.11016E+05	-0.98653E+03	0.43239E+06	-0.26496E+06	
13	0.13428E+07	0.83642E+06	0.34580E+05	-0.37490E+05	0.78905E+05	-0.13409E+06
	0.15710E+06	-0.15065E+05	0.13965E+05	-0.16230E+07	-0.76023E+06	
14	0.44438E+06	-0.96478E+06	0.29003E+06	-0.59136E+06	0.84566E+06	-0.98155E+06
	0.77988E+06	-0.59114E+06	0.76587E+06	0.12914E+07	0.21959E+07	
15	-0.10513E+07	0.81948E+06	0.31737E+06	-0.47745E+06	0.11280E+07	-0.20800E+07
	0.31680E+07	-0.74510E+06	0.17796E+07	-0.38980E+07	-0.18837E+06	

VITA

Prabh Nandan Singh was born on August 26, 1946, in Amritsar, Panjab, India. He attended G.R.D. Khalsa Higher Secondary School at Amritsar and passed in June, 1962. He then attended Panjab Engineering College, Chandigarh, India, and graduated in August, 1967, with a Bachelor of Science degree in Mechanical Engineering. After a few years in industry he was admitted to the Graduate School of Louisiana State University and is now a candidate for the Master of Science degree in Nuclear Engineering in August, 1972.